Lecture - Conductors

Charge can move in a conductor. This gives us a new way to describe currents and additional conditions on the static charge distributions on conductors.

Ohm's Law $\vec{J} = \sigma \vec{E}$ (sometimes true)

conductivity (σ) - Intrinsic property of conductor that controls conduction.

$$[\sigma] = \frac{S}{m} \qquad S \equiv \text{Siemen}$$

Archaic $1S = 1\,\text{mho} = 1\mho$

resistivity (ρ) - $\rho \equiv \frac{1}{\sigma}$ Intrinsic property

$$[\rho] = \Omega \cdot m$$

Resistance (R) -

$$R \equiv \frac{\Delta V}{I}$$

$$[R] = \Omega \equiv \frac{V}{A} \qquad \Omega = \text{ohm}$$

<u>Power</u> The power transferred from the field to the current is

$$P = \vec{F} \cdot \vec{v} = Q \vec{E} \cdot \vec{v}$$

or the power per unit volume

$$\frac{dP}{dv} = \rho \vec{v} \cdot \vec{E}$$

charge density

$$\vec{J} = \rho \vec{v}$$

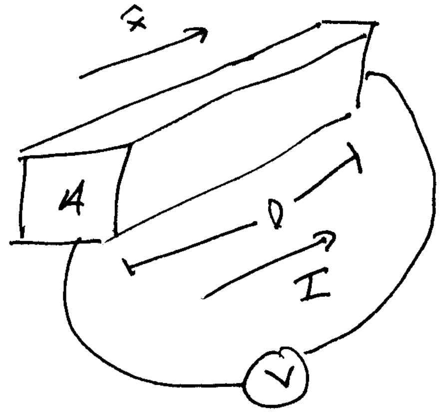
$$\frac{dP}{dv} = \vec{E} \cdot \vec{J}$$

The power dissipated in a material is

$$\frac{dP}{dv} = \vec{E} \cdot \vec{J} \qquad (\vec{E} \parallel \vec{J})$$

$\Rightarrow$ Joule Heating

Consider a block of material with cross-section A and length ℓ. A potential is established across a material that causes a current to flow



The measured resistance is $R = \frac{\Delta V}{I}$

Assume a constant current flows, $\vec{J} = J_0 \hat{x}$

By Ohm's Law, $\sigma \vec{E} = J_0 \hat{x}$.

The potential difference is

$$|\Delta V| = \left| -\int \vec{E} \cdot d\vec{\ell} \right| = \frac{J_0 \ell}{\sigma}$$

The total current is

$$I = \int \vec{J} \cdot d\vec{a} = I = J_0 A$$

Resistance

$$R = \frac{\Delta V}{I} = \frac{J_0 \ell / \sigma}{J_0 A} = \frac{\ell}{A \sigma}$$

or $$R = \frac{\rho \ell}{A}$$

Power

$$\frac{dP}{dv} = \vec{E} \cdot \vec{J} = \frac{J_0^{\,2}}{\sigma}$$

Total Power

$$P = \left(\frac{dP}{dv}\right) V = \frac{J_0^{\,2}}{\sigma} A \ell$$

$$= (J_0 A) \left(\frac{J_0 \ell}{\sigma}\right)$$

$$P = I \Delta V$$

Ex Same system, now $\sigma(x) = \gamma(x + x_0)$

$\Rightarrow$ conductivity changes with position.

$\Rightarrow$ How would you get this?

Again, assume steady state where $\vec{J} =$ constant.

$$\vec{J} = J_0 \hat{x}$$

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{J_0}{\gamma(x+x_0)} \hat{x}$$

$$|\Delta V| = \left| -\int \vec{E} \cdot d\vec{\ell} \right| = \left| -\int_0^{\ell} \frac{J_0}{\gamma(x+x_0)} dx \right|$$

$$= \frac{J_0}{\gamma} \ln\left(\frac{\ell + x_0}{x_0} \right)$$

$$I = J_0 A$$

$$R = \frac{|\Delta V|}{I} = \frac{1}{\gamma A} \ln\left(\frac{\ell + x_0}{x_0} \right)$$

This system has an addition feature.

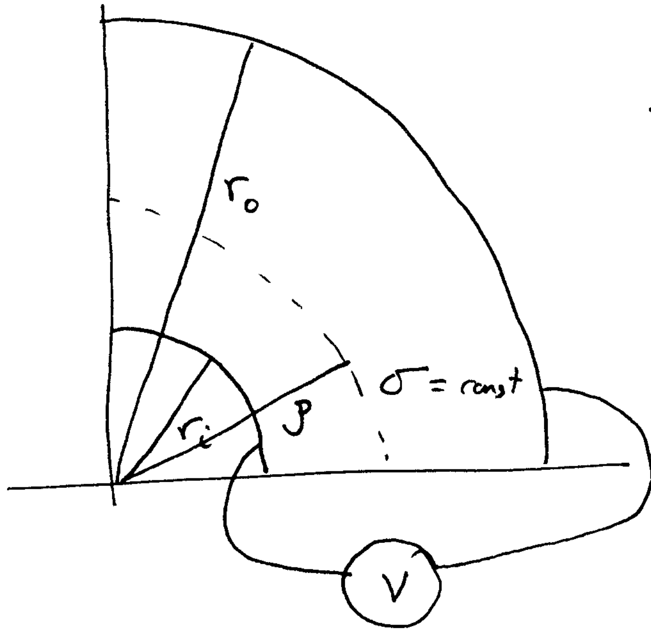
$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\rho = \varepsilon_0 \nabla \cdot \vec{E} = \varepsilon_0 \frac{d}{dx} \frac{J_0}{\gamma} (x + x_0)^{-1}$$

$$= - \frac{J_0 \varepsilon_0}{\gamma (x + x_0)^2}$$

$\Rightarrow$ A net charge density.

Ex



Thickness a

Again, $\vec{J}$ through any cross-section must be constant.

At a distance ρ, the cross-sectional area is

$$A(\rho) = \frac{\pi}{2} \rho a$$

If a current I flows through r_i, then the current density at r_i is

$$I = J_0 A(r_i)$$

Likewise, $J(\rho) = \dfrac{I}{A(\rho)}$

The field is then

$$E(\rho) = \frac{J(\rho)}{\sigma} = \frac{I}{\sigma A(\rho)}$$

$$E(\rho) = \frac{2I}{\sigma \pi a \rho}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{l} = -\int_{r_i}^{r_o} \frac{2I}{\sigma \pi a \rho} d\rho$$

$$= \frac{2I}{\sigma \pi a} \ln\left(\frac{r_o}{r_i}\right)$$

$$R = \frac{\Delta V}{I} = \frac{2}{\sigma \pi a} \ln\left(\frac{r_o}{r_i}\right)$$

Power

$$\frac{dP}{dv} = \vec{E} \cdot \vec{J} = \frac{J^2(\rho)}{\sigma} = \frac{I^2}{\sigma A^2}$$

$$\text{Power} = \int \frac{dP}{dv} dv = \frac{I^2 a}{\sigma} \int_0^{\pi/2} d\phi \int_{r_i}^{r_o} \rho \, d\rho \, \frac{dP}{dv}$$

$$= \frac{\pi}{2} a \frac{I^2}{\sigma} \int_{r_i}^{r_o} \rho \, d\rho \frac{1}{A^2} = \frac{\pi}{2} a \frac{I^2}{\sigma} \int_{r_i}^{r_i} d\rho \frac{\rho}{(\frac{\pi}{2}\rho a)^2}$$

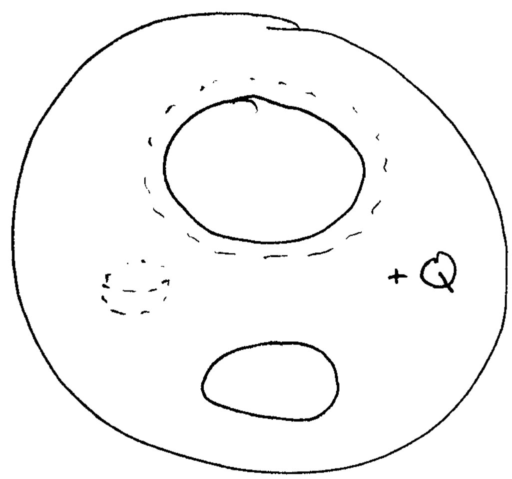
$$= \frac{2 I^2}{\pi a \sigma} \int_{r_i}^{r_o} \frac{d\rho}{\rho}$$

$$= \frac{2 I^2}{\pi a \sigma} \ln\left(\frac{r_o}{r_i}\right)$$

$$= I^2 R$$

Electrostatics in Conductors

Consider a conductor that has a charge Q spread throughout it in some manner.



Where can the charge be?

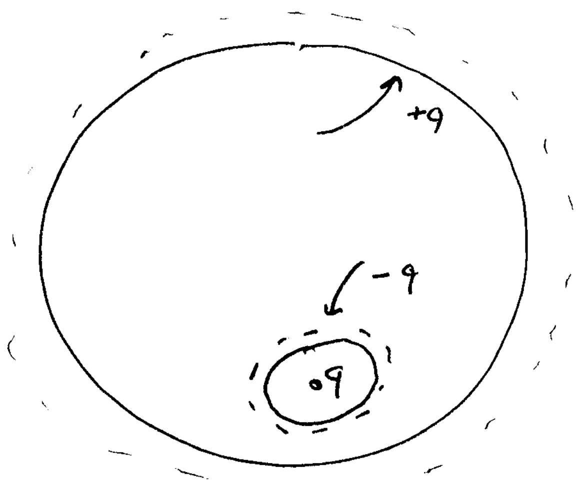
I. Inside the metal – Use a small Gaussian surface around charge, by Gauss' there must be a field at the Gaussian surface. Field $\Rightarrow$ current $\Rightarrow$ losses. A current will flow until net charge is removed.

II. Inner cavities – Once again the electric field in the conductor must be zero

$\Rightarrow$ charge in Gaussian surface $= 0$

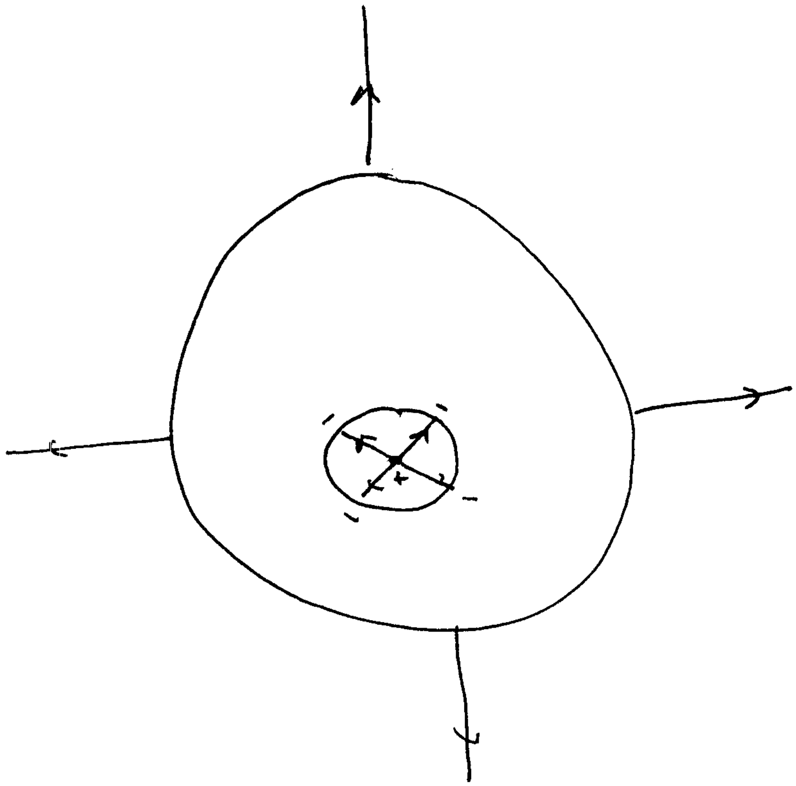
III - That leaves the outer surface.
Net charge on a conductor is at the outer surface.

Now, place a net charge in cavity.

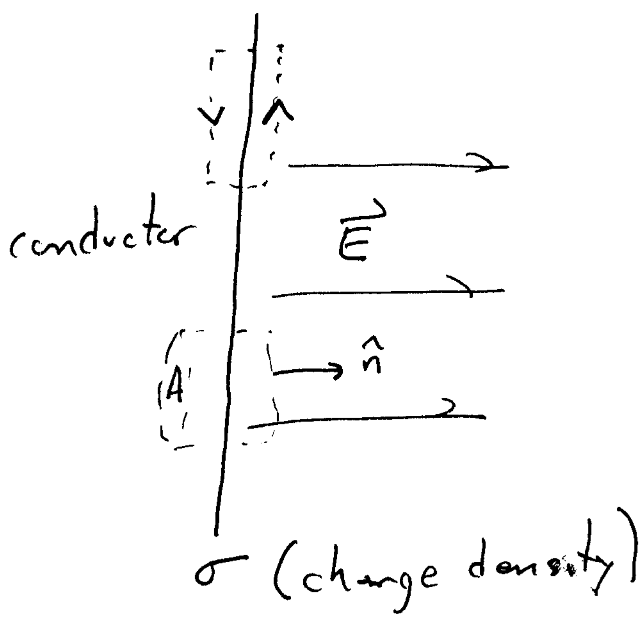


I. Use Gaussian surface around cavity, $\vec{E} = 0$
$\Rightarrow Q_{enc} = 0 \Rightarrow$ There must be $-q$ charge distributed around inner surface.
$\Rightarrow$ If q is in the center, we could calculate the density

II. Use GS around full conductor
$\Rightarrow Q_{enc} = q \Rightarrow +q$ on outer surface.



Consider an arbitrary conducting surface in an electric field.



I. Use Gaussian Pill box at the surface

$$(\vec{E}\cdot\hat{n})A = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

Let $|\vec{E}_n| \equiv \vec{E}\cdot\hat{n}$, the normal component of the field

$$\sigma = \varepsilon_0 |\vec{E}_n|$$

II. Use Stokesian Path at Surface

Let $|\vec{E}_t| = \vec{E}\cdot\hat{t}$ where $\hat{t}$ tangent to the surface.

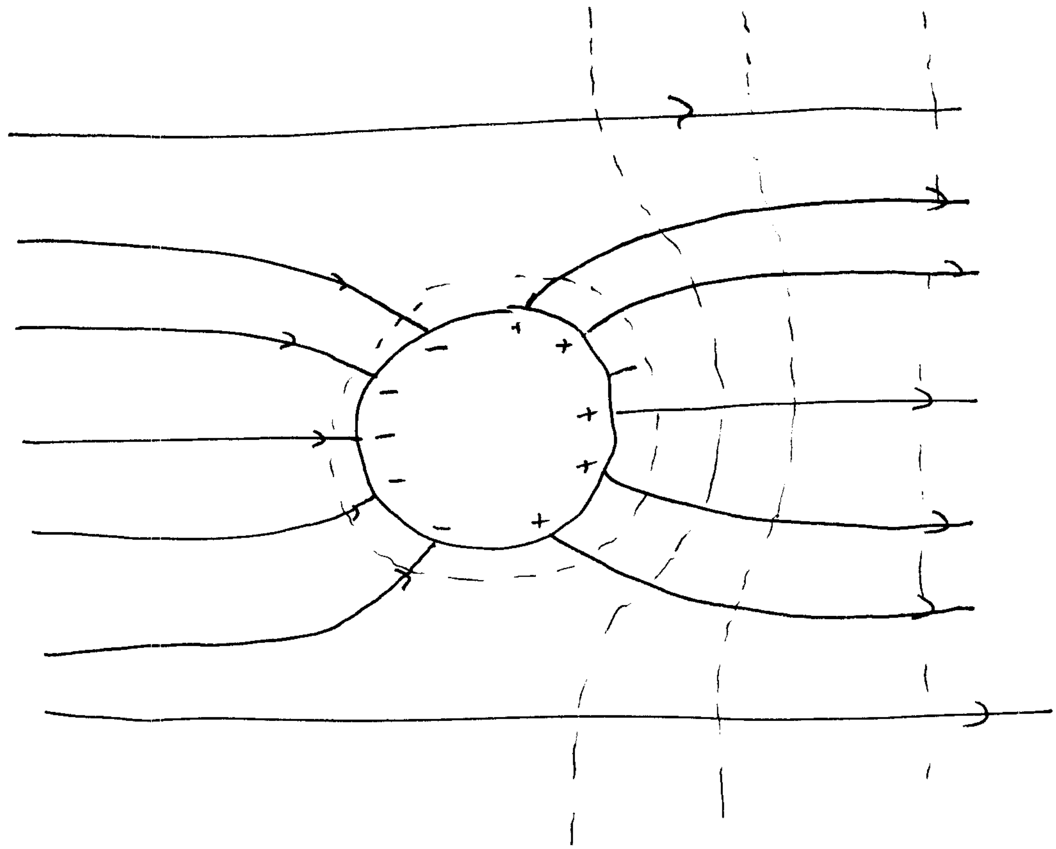
$$\vec{E}\cdot\hat{t}\,l = 0 \qquad \text{since} \quad \nabla\times\vec{E} = 0$$

So the tangential component of the field is zero

$\Longrightarrow$ The field is normal to a conductors surface

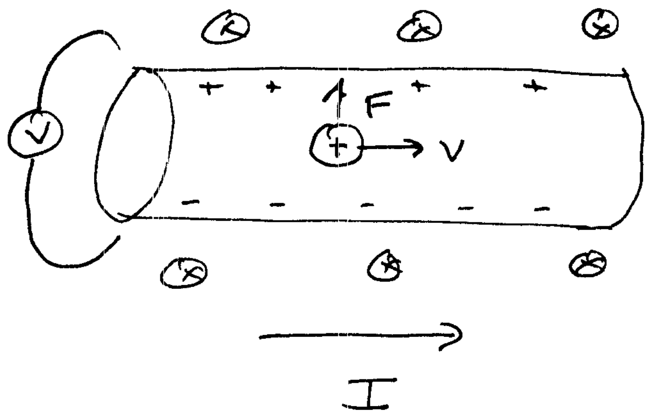
$\Longrightarrow$ The conductors surface is an equipotential.

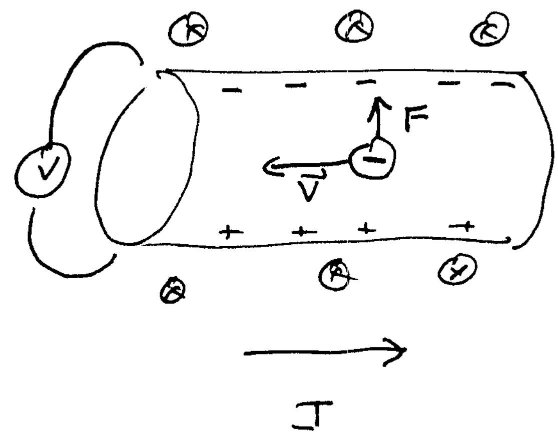
⑤



Hall Effect

– Let $\vec{B}$ point into page





$\Rightarrow$ The sign of the voltage changes with the charge carrier sign.

For equilibrium to be achieved, the magnetic force must balance the electric force.

$$q\vec{E} = q\vec{v} \times \vec{B}$$

or $E = vB$

The current density is $J = \rho v = qNv$
where N = charge carrier density.

$$E_H = \frac{JB}{Nq} \equiv R_H JB$$

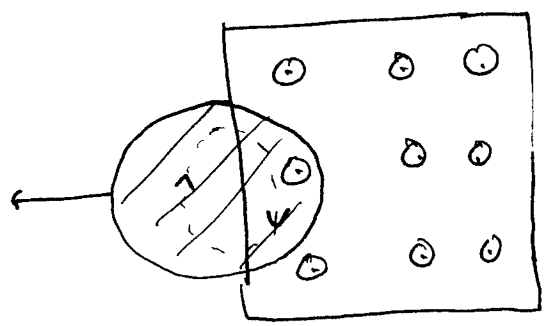
Hall Coefficient

$$R_H = \frac{1}{Nq}$$

Aluminum $\quad R_H = -3 \times 10^{-11} \ \Omega m / T$

Zinc $\quad R_H = +3.3 \times 10^{-11} \ \Omega m / T$

Eddy Currents

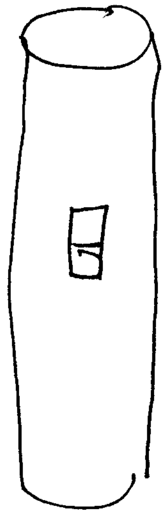


Φ is changing as conductor enters field.

Changing flux causes an emf that drives a current.

⟹ The current produces losses in the metal, kinetic energy is lost.

⟹ Magnetic braking



Magnet falls through tube with greatly decreased velocity $\Rightarrow$ lower kinetic energy

Where did lost $\Delta U = mgh$ go? Heating caused by currents in metal.

<u>Conduction</u> Let v_t be the thermal velocity of charges in a metal, l be the mean distance between collisions, and τ the mean time between collisions.

$$\tau = \frac{l}{v_t}$$

The average velocity (in the direction of current)

$$v_{ave} = \frac{1}{2} a \tau$$

average over velocities between collisions.

$$= \frac{1}{2} \frac{a\, l}{v_t}$$

If the free charge density is N and the charge of a charge carrier is q

$$J = N q\, v_{ave} = \frac{N q\, l}{2 v_t} a$$

$$a = \frac{qE}{m}$$

$$J = \underbrace{\left(\frac{Nq^2 l}{2 m v_t} \right)}_{\sigma} E$$

$\Rightarrow$ Drude model - It has the right stuff but QM warps v_t

Fermi Velocity

copper

Temperature 10^5 K

Energy $7\,eV = 1.1 \times 10^{-18}$ J

$= \frac{1}{2} m v^2$

$v = 1.5 \times 10^6$ m/s

Much higher than classical energy.