Electric Displacement

The bound charge ρ_b is just charge that is there because a polarization changes with position.

If it is, the only charge around, Gauss' Law is

$$\nabla \cdot \vec{E} = \rho_b / \varepsilon_0$$

If there are other charges around, called free charges, the total charge is

$$\rho = \rho_f + \rho_b$$

and Gauss' Law becomes

$$\nabla \cdot \vec{E} = \rho_f / \varepsilon_0 + \rho_b / \varepsilon_0$$

$$\varepsilon_0 \nabla \cdot \vec{E} = \rho_f + \rho_b$$

or using $\nabla \cdot \vec{P} = - \rho_b$

$$\varepsilon_0 \nabla \cdot \vec{E} = \rho_f - \nabla \cdot \vec{P}$$

$$\nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = \rho_f$$

<u>Electric Displacement</u> $(\vec{D})$

$$\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P}$$

<u>Gauss' Law</u>

$$\nabla \cdot \vec{D} = \rho_f$$

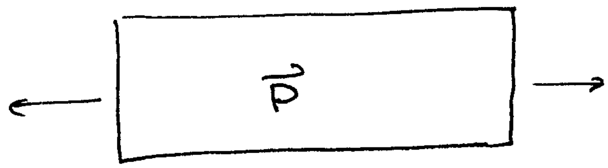
$\Rightarrow$ We can pull all those cool Gauss' law tricks with the Displacement.

$\Rightarrow$ <u>Caveat</u>

$$\nabla \times \vec{D} = \varepsilon_0 \nabla \times \vec{E} + \nabla \times \vec{P} \qquad \text{(static)}$$

$$= \nabla \times \vec{P} \neq 0 \text{ generally}$$

Let's start with our polarized slab,



$\rho_f = 0 \qquad \sigma_f = 0$

$\nabla \cdot \vec{D} = 0$ everywhere

$\vec{D} = \text{constant}$

Reflection $\rightarrow$ $\vec{D} = 0$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

<u>Inside</u>

$$\vec{E} = -\frac{\vec{P}}{\varepsilon_0}$$

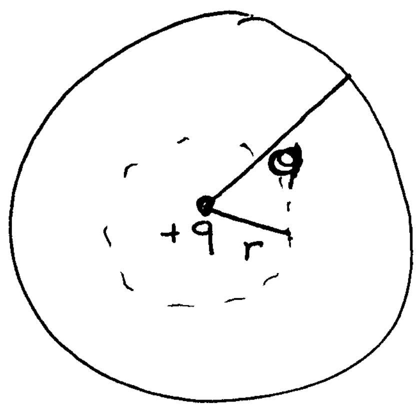
<u>Outside</u>

$\vec{E} = 0$ (since $\vec{P} = 0$)

which is what we found before.

Now suppose we have a medium that produces a polarization $\vec{P}$ in response to a electric field. We don't know how yet.

<u>Ex</u> Point charge q imbedded in center of spherical dielectric. Compute $\vec{D}$ everywhere, and $\vec{E}$ everywhere you can.



$$\nabla \cdot \vec{D} = \rho_f$$

$$\Phi_d \equiv \oint \vec{D} \cdot d\vec{a} = \int \rho_f \, dv = q$$

$$4\pi r^2 D(r) = q$$

$$\vec{D}(r) = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$

both inside and outside.

For $r > a$, $\vec{P} = 0$ and

$$\varepsilon_0 \vec{E} + \vec{P} = \vec{D} \Rightarrow \vec{E} = \vec{D}/\varepsilon_0$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Inside $(r < a)$ we need to know the polarization so we cannot say anything about the electric field.

Dielectrics - (Insulators) - The response to an applied field could be anything. Not uncommon.

Linear Dielectrics Polarization $\propto$ Field

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

Electric Susceptibility (χ_e)

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E}$$

$$= \varepsilon_0 (1 + \chi_e) \vec{E}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\varepsilon = \varepsilon_0 (1+\chi_e)$$

- Permittivity (ε) - Permittivity of material.

- Relative Permittivity (ε_r)

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

$\Rightarrow$ Dielectric Constant (UP II)

$$K = \varepsilon_r$$

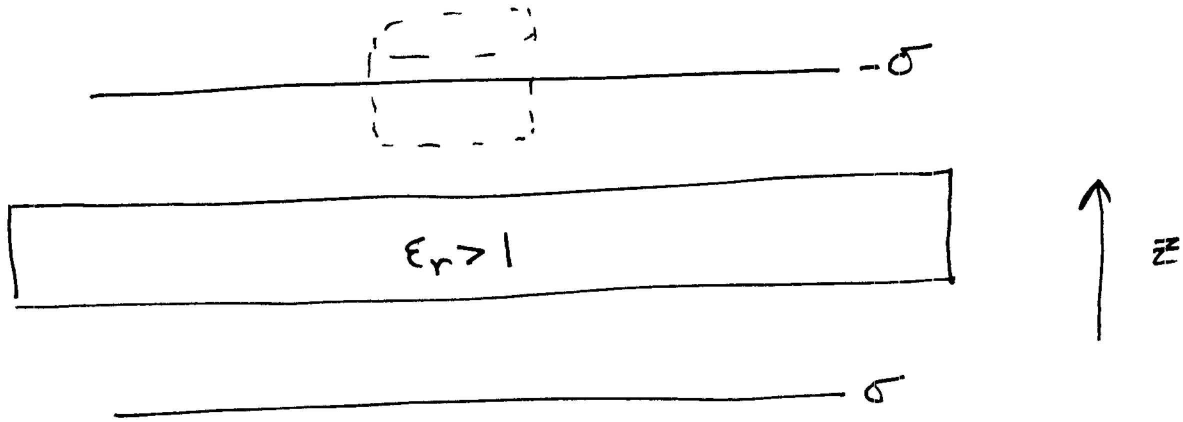
Let's return to our point charge embedded in a polarizable medium. If that medium is a linear dielectric,

Inside $\vec{D} = \frac{q}{4\pi r^2}\hat{r} = \varepsilon \vec{E}$

$$\vec{E} = \frac{q}{4\pi\varepsilon r^2}\hat{r} = \frac{q}{4\pi\varepsilon_0(1+\chi_e)r^2}\hat{r}$$

$$= \frac{q}{4\pi\varepsilon_0 K r^2}\hat{r}$$

Let's return to our old friend the parallel plate capacitor —



<u>Outside</u>

$\vec{D}_o = 0$ $\quad\quad$ $\vec{P}_o = 0$ $\quad\quad$ $\vec{E}_o = 0$

$\cancel{D_i \angle \epsilon}$

<u>Inside</u>

$$\Phi = -D_i A = \frac{Q_{enc}}{\cancel{\epsilon_0}} = -\sigma A$$

$$D_i = +\sigma$$

$$\vec{D}_i = \sigma \hat{z}$$

<u>Linear Dielectric</u>

$$\vec{D}_i = \epsilon \vec{E}_i$$

$$\vec{E}_i = \frac{\sigma}{\epsilon} \hat{z} \quad \text{(Anywhere between plates)}$$

<u>Inside Dielectric</u> $\varepsilon_r = (1+\chi_e) > 1$

$$\vec{E}_{id} = \frac{\sigma}{\varepsilon_0 \varepsilon_r}\hat{z}$$

$$\vec{P}_{id} = \varepsilon_0 \chi_e \vec{E} = \left(\frac{\chi_e}{1+\chi_e}\right)\sigma\hat{z}$$

<u>Bound charge</u>

$$\rho_f = -\nabla\cdot\vec{P} = 0$$

$$\sigma_b = \vec{P}\cdot\hat{n} = \frac{\chi_e}{(1+\chi_e)}\sigma$$

<u>Outside Dielectric</u> (But between plates)

$\varepsilon_r = 1 \qquad \varepsilon = \varepsilon_0$

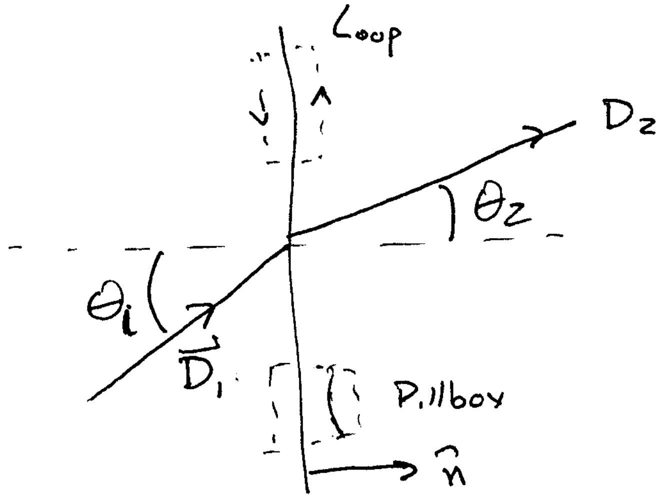
$$\vec{E}_{io} = \frac{\sigma}{\varepsilon_0}\hat{z}$$

Electrostatic Boundary Conditions

Without Dielectrics

$$\vec{E}_2 \cdot \hat{n} - \vec{E}_1 \cdot \hat{n} = \frac{\sigma}{\varepsilon_0}$$

$$\vec{E}_2 \cdot \hat{t} = \vec{E}_1 \cdot \hat{t}$$



Gaussian Pillbox

$$\vec{D}_2 \cdot \hat{n} - \vec{D}_1 \cdot \hat{n} = \sigma_f$$

If $\sigma_f = 0$, $\quad D_1 \cos\theta_1 = D_2 \cos\theta_2$

Stokesian Loop

$$\oint \vec{E} \cdot d\vec{l} = 0 \qquad \vec{E}_1 \cdot \hat{t} = \vec{E}_2 \cdot \hat{t}$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

$$\frac{E_1}{D_1} \tan\theta_1 = \frac{E_2}{D_2} \tan\theta_2$$

$$D_1 = \varepsilon_0 \varepsilon_{r1} E_1$$

$$\frac{1}{\varepsilon_{r1}} \tan\theta_1 = \frac{1}{\varepsilon_{r2}} \tan\theta_2$$

So $\vec{D}$ bends at a dielectric interface.

A new current,

$$\frac{\partial \rho_b}{\partial t} = -\nabla \cdot \left(\frac{\partial P}{\partial t} \right) = -\nabla \cdot \vec{J}_{pol}$$

<u>Polarization Current</u>

$$\vec{J}_{pol} = \frac{\partial \vec{P}}{\partial t}$$

<u>Displacement Current</u> (new)

$$\vec{J}_d = \underbrace{\varepsilon_0 \frac{\partial \vec{E}}{\partial t}}_{\text{old}} + \frac{\partial \vec{P}}{\partial t} = \frac{\partial \vec{D}}{\partial t}$$