Lecture 1 - Overview

Discuss Policy / Syllabus

How to prepare? Work throughout the week.

Prepare your own notes.

Work additional problems, Odd # + Griffiths.

Give your notes as a lecture.

Work as independently as possible

Use my help wisely.

Overview We will be working with electric and magnetic fields in free space until test 1 is given. We will also be working with fixed distributions of charge and current. We will develop Maxwell's equations from the div, grad, curl perspective. After Test 1, we will introduce materials into the fields and learn how to deal with the currents and charges that are produced. We will also re-introduce to concept of potential.

After Test 2, we will more thoroughly explore the topic of radiation.

We will work with physical laws that have three different forms.

Next Week Gauss

$$\oint_S \vec{E} \cdot \hat{n}\, da = \frac{Q_{enc}}{\varepsilon_0} \quad \text{or} \quad \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{E} = \frac{kq}{r^2}\hat{r} \qquad \text{Coulomb}$$

For systems where all charge or all current is constant, we can

(1) In situations with sufficient symmetry, use the integral form to calculate the field.

(2) In other situations, calculate the field by direct integration.

When a material is introduced into the field, charge densities and currents are induced. These new sources add to the field and we have to find solutions that are self-consistent, where induced fields induce the correct charges. Outside of a few tricks (very good tricks) we will solve the differential form of the law to manage these situations.

For those of you working on A1, you may save tons of work by remembering $V = \frac{kq}{r}$ and

$$\vec{E} = -\frac{dV}{dr} = -\nabla V$$

Vectors

A scalar has a magnitude but no direction.

$\Rightarrow$ A number

A vecter has magnitude and direction.

$\Rightarrow$ Quantity that transforms in the same way as the position vector.

$\Rightarrow$ Quantity that does not change form under rotation.

We will represent vecters as 3-tuples

$$\vec{A} = (A_x, A_y, A_z)$$

The above just means that not all ~~vectors~~ 3-tuples are vectors.

For example, (3 pigs, 1 dog, 5 moose) does not make sense as we rotate it through 45°.

Symmetry - A symmetry of a system is a transformation that leaves the system unchanged.

$\Rightarrow$ Ex A rotation about the origin will leaves a system with spherical symmetry unchanged.

$\Rightarrow$ When you invoke symmetry, you must tell what symmetry is actually being used.

Transformations

One vector (or coordinate system) can be transformed into another by multiply by the appropriate matrix $\overleftrightarrow{T}$

$$\vec{A}' = \overleftrightarrow{T}\,\vec{A}$$

$$\overleftrightarrow{T} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$$

For example, to transform to a frame rotated by an angle θ about the z-axis.

$$\overleftrightarrow{T} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Returning to dfn of vector, a quantity $\vec{A}$

$$\vec{A} = f(x,y)\,\hat{x} + g(x,y)\,\hat{y}$$

is a vector if when transformed to another frame, which we will call the primed frame

$$\vec{A}' = f(x',y')\,\hat{x}' + g(x',y')\,\hat{y}'$$

$\Rightarrow$ f, g same function.

<u>Ex</u> Is the position vector $\vec{r} = (x, y, z)$ a vector?

$A_x = x$, $A_y = y$, $A_z = z$

$f(x,y) = x$ $\quad g(x,y) = y$

$$\begin{pmatrix} A_x' \\ A_y' \\ A_z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$A_x' = \cos\theta\, A_x + \sin\theta\, A_y = x\cos\theta + y\sin\theta$$

$$A_y' = -\sin\theta\, A_x + \cos\theta\, A_y = -x\sin\theta + y\cos\theta$$

$$A_z' = A_z = z$$

In the prime coordinate,

$$x' = x\cos\theta + y\sin\theta = A_x'$$

$$y' = -x\sin\theta + y\cos\theta = A_y'$$

$$z' = z = A_z'$$

Other more substantial examples given in book.

We can write the multiplication we just did as

$$A_i' = \sum_j T_{ij} A_j \qquad j = x, y, z \text{ or } 1, 2, 3$$

To save a little writing, we will use a summation convention where the sum is implied for a repeated index

$$\sum_j T_{ij} A_j = T_{ij} A_j$$ because j is used twice.

Vecter Products

Dot Product

$\vec{A} \bullet \vec{B} = \text{scalar}$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = |\vec{A}||\vec{B}| \cos\theta$$

- Dot $\perp$ vectors $\rightarrow 0$

Cross-Product

$\vec{A} \times \vec{B} = \vec{C}$

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta \Rightarrow$$ Volume of parallelagram formed by $\vec{A}, \vec{B}$

$$\vec{C} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
- $\vec{A} \times \vec{A} = 0$

Unit Vecters

$\hat{A}$ is a vecter of unit length in the direction of $\vec{A}$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

Vecter Modulus or Length - $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Triple Products

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$$

$$= \det \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$\Rightarrow$ Volume of parallel-piped formed by $\vec{A}, \vec{B}, \vec{C}$.

BAC-CAB Rule

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - C(\vec{A}\cdot\vec{B})$$

$$\neq (\vec{A}\times\vec{B})\times\vec{C}$$

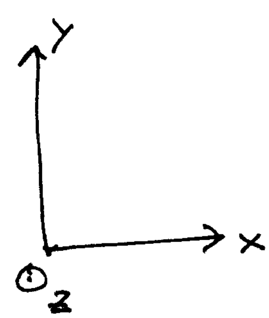
$\Rightarrow$ Not associative

$\Rightarrow$ $\vec{A}\times\vec{B}\times\vec{C}$ has no meaning

Ex Proof by counter example

$$(\hat{y}\times\hat{x})\times\hat{x} = -\hat{z}\times\hat{x} = -\hat{y}$$

$$\hat{y}\times(\hat{x}\times\hat{x}) = 0$$



Operators

"del" (nabla)

$$\nabla \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

Gradient (scaler → vector)

$$\text{grad } f \equiv \nabla f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

Divergence (vector → scaler)

$$\nabla \cdot \vec{A} \equiv \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Curl (vector → vector

$$\text{curl } \vec{A} = \nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Laplacian

$$\nabla^2 \cancel{A} f = \nabla \cdot \nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

- The divergence and curl must be applied to a vector, $\nabla \cdot f$ does not make sense.

- The Laplacian ~~and the gradient~~ may be applied to a vector or a scaler.

$$\nabla^2 \vec{A} = \left(\nabla^2 A_x, \nabla^2 A_y, \nabla^2 A_z \right)$$

Working with Operators

- ∇, $\nabla \cdot$, $\nabla \times$, ∇^2 are operators.
- In general, operators do not commute.

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ does not imply

$$\vec{r} \cdot \nabla = \nabla \cdot \vec{r}$$

$$\vec{r} \cdot \nabla = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} + z \frac{\partial}{\partial z}$$

$$\nabla \cdot \vec{r} = 3$$

A few identities, for more examine the front inside cover.

$$\nabla(fg) = f\nabla g + g\nabla f$$
$$= (\nabla g)f + (\nabla f)g$$

$$\nabla\left(\frac{f}{g}\right) = \frac{1}{g}\nabla f - \frac{f}{g^2}\nabla g$$

$$\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$$

$$\nabla\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B}$$
$$+\vec{A}(\nabla\cdot\vec{B}) - \vec{B}(\nabla\cdot\vec{A})$$

etc.

- $\nabla\times(\nabla f) = 0 = (\nabla\times\nabla)f$
- $(\nabla\times\nabla)\cdot\vec{A} = 0$
- $(\nabla\times\nabla)\times\vec{A} = 0$
- $\nabla\cdot(\nabla\times\vec{A}) = 0$

Important

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$