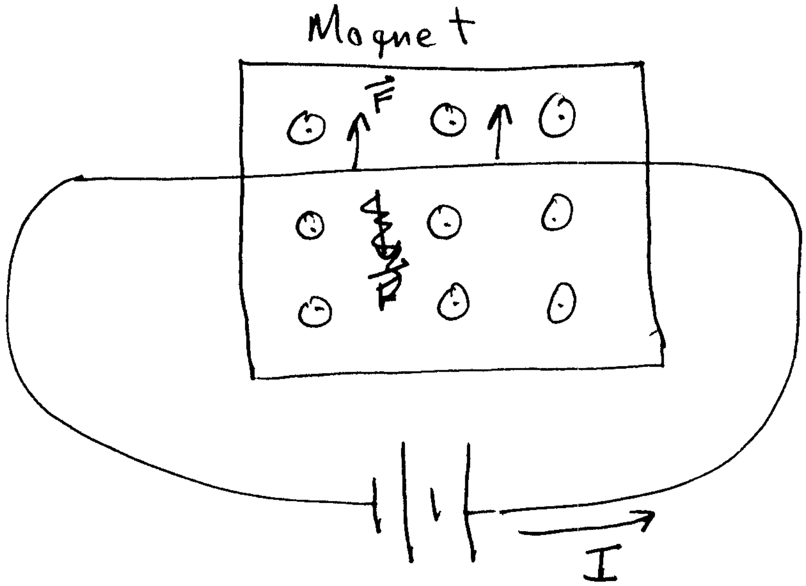
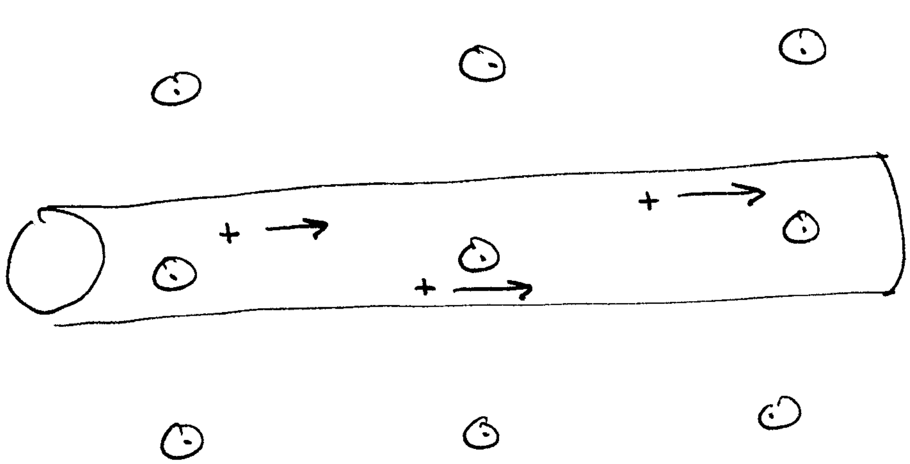
A tricky bit

Consider the following experiment, which most of you did in UPII.

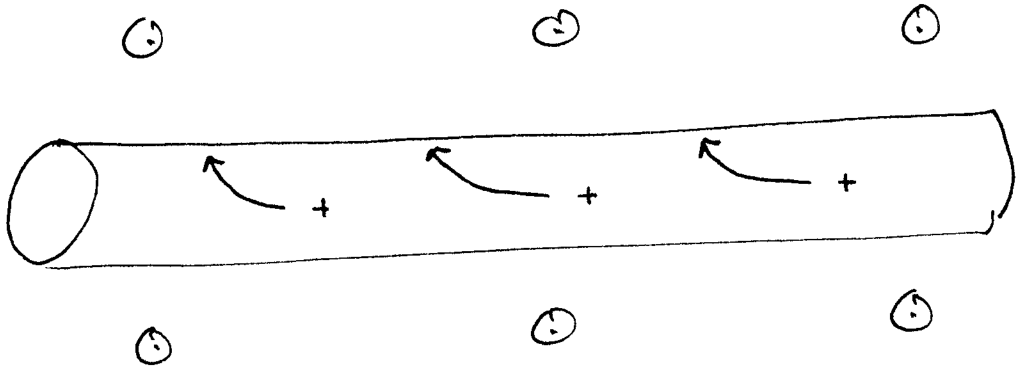


The wire jumps out of the magnet, that is the wire accelerates and the magnetic field appears to do work on the wire.

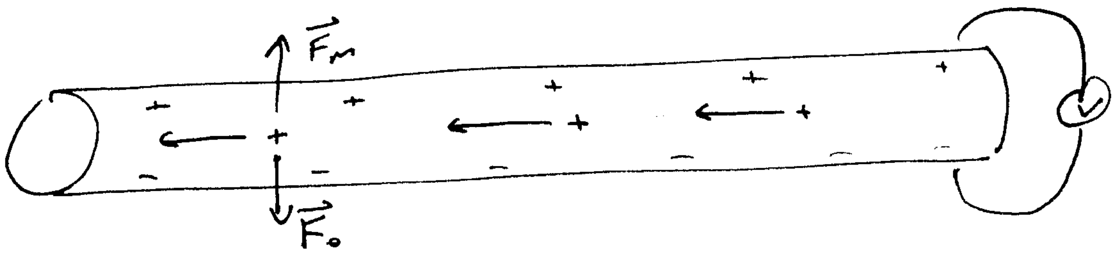
Let's Look Closer



The magnetic field causes the charges to move in circular orbits.



Excess charge builds up until the magnetic force is balanced by an electric force.



This produces a measurable potential difference across the sides of the wire (Hall Effect).

Now, this is no problem if the wire is stationary because no work is being done. However, if the wire moves in response to the magnetic force, we have something to explain.

Let $\vec{w}$ be the velocity of the charge carrier in the stationary wire. The current is then

$$\vec{I} = \lambda \vec{w}$$

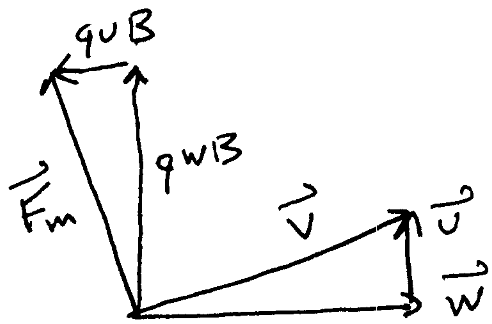
where λ is the charge per unit length.

If the wire moves with velocity $\vec{u}$, then the total velocity of a charge in the moving wire is

$$\vec{v} = \vec{w} + \vec{u}$$

and the magnetic force on the charge

$$\vec{F}_m = q\vec{v} \times \vec{B} = q\vec{w} \times \vec{B} + q\vec{u} \times \vec{B}$$



Now, $\vec{F}_m$ has a component along the current, opposing the flow of current. In time dt, the charge moves a distance $w\,dt$ along the wire. The battery must do additional work $F w\,dt = quBw\,dt$. If there are λ charge carriers per unit length and the length of the wire is a, then the total work in time dt is

$$W = (\lambda a) u B w \, dt$$

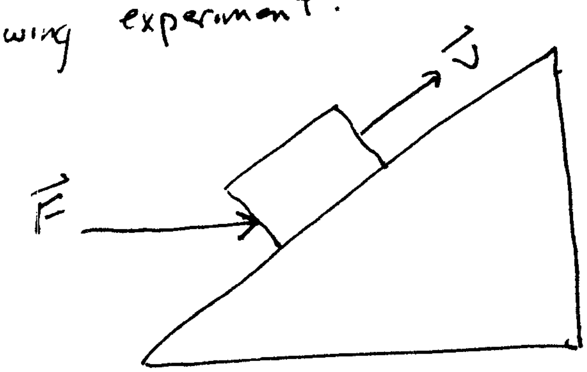
where λa = total charge.

The current in the wire is λw, so the work done by the battery is

$$W = I B u a \, dt = (I B a) \cdot \text{distance}$$

So the work we attributed to the magnetic field $F_m \cdot$ distance was actually the additional work of the battery.

The magnet redirects the force of the battery. The magnet plays the role of the normal force in the following experiment.



Normal forces do ~~not~~ no work, but redirect $\vec{F}$ so it can do work.