# PHYS 3414 - Electricity and Magnetism- Final Exam 1

All problems are worth 25 points. Turn in solutions to four of the six problems to be graded. If you turn in more than four solutions, I will grade the first four. You are allowed to drop one-half of a test, so I will take the first two problems turned in as the first half-test and the second two problems turned in as the second half-test.

**1** A solenoid ($N'$ turns per unit length) has current that is changing as $I(t) = I_0 e^{(t/\tau)}$. A single loop of wire is inside the solenoid. The axis of the loop is parallel to the axis of the solenoid. The loop has radius $a$, cross-sectional area $A$ and conductivity $\sigma$. Compute the current in the wire.

**2** A spherically symmetric varying volume charge density fills the space $r > a$. There is no charge in the region $r < a$. The charge density varies as $\rho = \rho_0 / r^4$. Compute the electric field everywhere.

**3** A solenoid ($N'$ turns per unit length) carries a constant current $I$ and is partially filled with an iron core of radius $a$ and relative permeability $\mu_r$. Compute $\vec{B}$ and $\vec{H}$ at points inside the solenoid. Report a value for both inside and outside of the iron. Compute the surface current density $K_b$ on the iron.

**4** A ring with radius $a$ and constant linear charge density $\lambda$ lies in the $x - y$ plane centered at the origin. Compute the electric potential at a point $R$ along the $z$-axis.

**5** The potential on the surface of an infinite cylinder is given by $V(a, \phi) = V_0(\sin(\phi) + \cos(\phi))$. Find the potential at all points inside the cylinder and the field at the origin.

**6** Two straight wires carry a current $I$ in the $+\hat{y}$ direction. The wires run through the points $\pm \ell \hat{x}$. The wires have length $D$ and are centered on the $x$ axis. Compute the vector potential at the origin.

(1) The magnetic field of the solenoid is

$$B(t) = \mu_0 N' I(t)$$

The magnetic flux through the loop is

$$\Phi_m = B(t)\pi a^2$$

The emf around the loop (Faraday) is

$$\text{emf} = \frac{-d\Phi_m}{dT} = -\pi a^2 \mu_0 N' \frac{dI}{dT}$$

$$= \pi a^2 \mu_0 N' \frac{I_0}{\tau} e^{-t/\tau}$$

The resistance of the loop is

$$R = \frac{l}{\sigma A} = \frac{2\pi a}{\sigma A}$$

and by Ohm's law the current is

$$I_{loop} = \frac{\text{emf}}{R} = \frac{\text{emf}\cdot\sigma A}{2\pi a} = \frac{\sigma A}{2\pi a}\left(\pi a^2 \mu_0 N' \frac{I_0}{\tau} e^{-t/\tau}\right)$$

$$= \frac{1}{2}\frac{\sigma A a \mu_0 N' I_0}{\tau} e^{-t/\tau}$$

(2) For a spherical system, the flux out of a spherical Gaussian surface of radius $r$ is

$$\Phi = 4\pi r^2 E(r) = \frac{Q_{enc}}{\varepsilon_0} \quad \text{(Gauss' Law)}$$

The charge inside the Gaussian surface is 0 if $r<a$, so $\vec{E}_{r<a} = 0$.

For $r>a$,

$$Q_{enc} = \int_a^r 4\pi r^2 \rho(r)\,dr$$

$$= \int_a^r \frac{4\pi r^2 \rho_0}{r^4}\,dr$$

$$= \int_a^r \frac{4\pi \rho_0}{r^2}\,dr$$

$$= -\frac{4\pi\rho_0}{r}\Big|_a^r = 4\pi\rho_0\left(\frac{1}{a} - \frac{1}{r}\right)$$

The electric field is then

$$\vec{E}_{r>a} = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{1}{\varepsilon_0 r^2}\left(\frac{1}{a} - \frac{1}{r}\right)\hat{r}$$

③ The $\vec{H}$ field inside the solenoid is

$$\vec{H} = N'I\,\hat{z} \qquad \text{if } \hat{z} \text{ is the axis.}$$

There are no free currents in the solenoid so this is the $\vec{H}$ field both inside and outside the iron.

The magnetic field outside the iron is

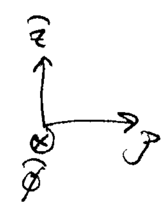
$$\vec{B}_0 = \mu_0 \vec{H} = \mu_0 N' I\,\hat{z}$$

Inside the iron,

$$\vec{B}_i = \mu_0 \mu_r \vec{H} = \mu_r \vec{B}_0$$

The magnetization in the iron is

$$\vec{M} = \chi_m \vec{H} = (\mu_r - 1)\vec{H}$$



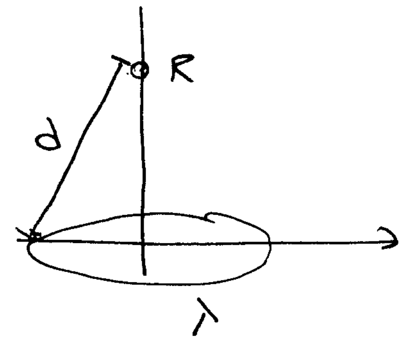

$$= (\mu_r - 1) N' I\,\hat{z}$$

The surface current is

$$\vec{K}_b = \vec{M} \times \hat{n} = \vec{M} \times \hat{\rho} = (\mu_r - 1) N' I\,\hat{z} \times \hat{\rho}$$

$$= (\mu_r - 1) N' I\,\hat{\phi}$$

(4)




All the charge is equidistant from the loop,

$$V = \frac{kq}{d}$$

$$d = \sqrt{R^2+a^2} \qquad q = 2\pi a \lambda$$

$$V = \frac{2k\pi a \lambda}{\sqrt{R^2+a^2}} = \frac{a\lambda}{2\varepsilon_0 \sqrt{R^2+a^2}}$$

(5) Cylindrical System $\rho < a$

$$V_i(\rho, \phi) = \sum_n A_n \rho^n \sin n\phi + B_n \rho^n \cos n\phi$$

At the border, $\rho = a$

$$V(a, \phi) = V_0 \sin\phi + V_0 \cos\phi$$

$$= \sum_n A_n a^n \sin n\phi + B_n a^n \cos n\phi$$

By orthogonality,

$A_n, B_n = 0$ if $n \neq 1$

$A_1 a = V_0$ $\qquad$ $B_1 a = V_0$

$$V(\rho, \phi) = \frac{V_0}{a} \rho \sin\phi + \frac{V_0}{a} \rho \cos\phi$$

The field is

$$\vec{E} = -\nabla V$$

$$= -\frac{\partial V}{\partial \rho}\hat{\rho} - \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{\phi}$$

$$= -\frac{V_0}{a}\left(\sin\phi + \cos\phi\right)\hat{\rho}$$

$$-\frac{V_0}{a}\left(\cos\phi - \sin\phi\right)\hat{\phi}$$

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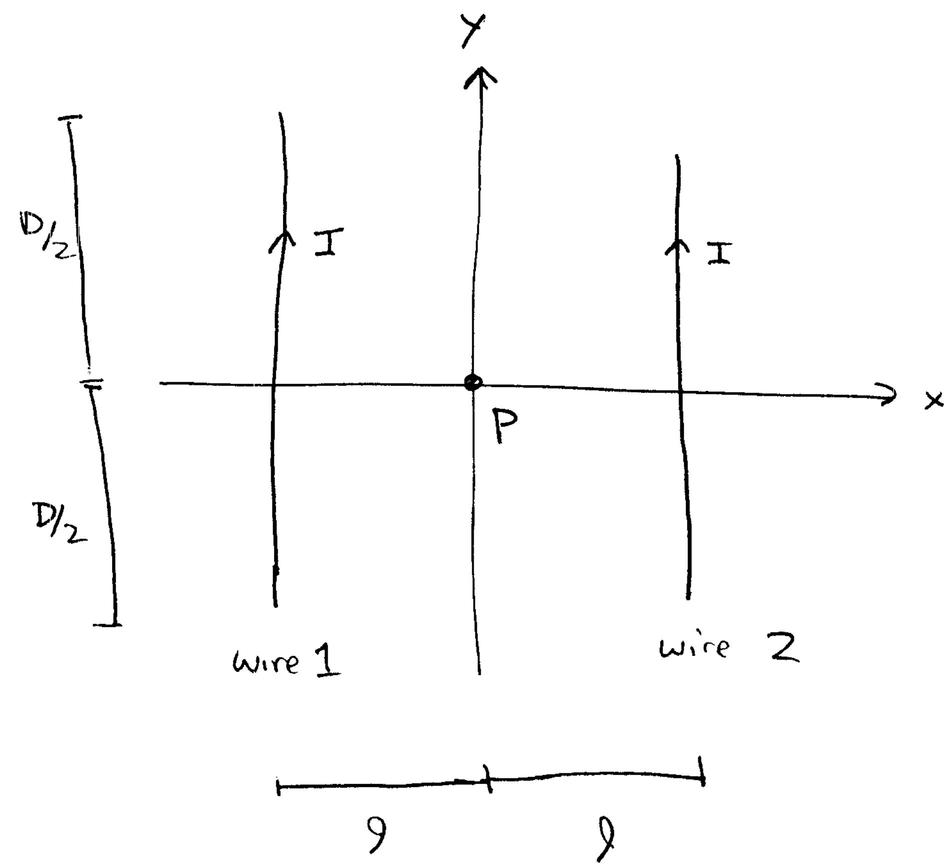


The vector potential is given by

$$\vec{A}_1 = \frac{\mu_0}{4\pi}\int \frac{d\vec{I}_1}{r}$$

$$= \frac{\mu_0 I \hat{y}}{4\pi}\int_{-D/2}^{D/2} \frac{dy}{\sqrt{l^2+y^2}}$$

Evidently $\vec{A}_1 = A_2$

Perform the integral,

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right) \quad \text{Schaum}$$

$$\vec{A}_1 = \frac{\mu_0 I \hat{y}}{4\pi} \ln\left(y + \sqrt{y^2+\ell^2}\right)\Big|_{-D/2}^{D/2}$$

$$= \frac{2\mu_0 I \hat{y}}{4\pi} \cdot \ln\left(y + \sqrt{y^2+\ell^2}\right)\Big|_{0}^{D/2}$$

$$= \frac{\mu_0 I \hat{y}}{2\pi} \ln\left(\frac{D/2 + \sqrt{(D/2)^2 + \ell^2}}{\ell}\right)$$

$$\vec{A} = \vec{A}_1 + \vec{A}_2 = \frac{\mu_0 I \hat{y}}{\pi} \ln\left(\frac{D}{2\ell} + \sqrt{1 + \left(\frac{D}{2\ell}\right)^2}\right)$$