PHYS 3414 - Electricity and Magnetism- Homework Set 4

Chapter 4 - Faraday's Law

Due 4:30pm Monday February 18, 2008.

Good's Problems

4.2

4.4 The current is zero.

4.6

4.8

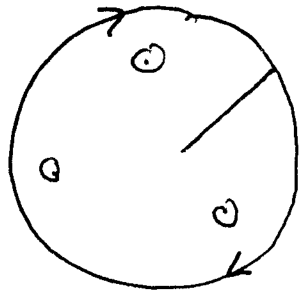
4.12

4.14

4.16

4.18

4.2



$a = 3\,\text{cm}$

$N = 170$

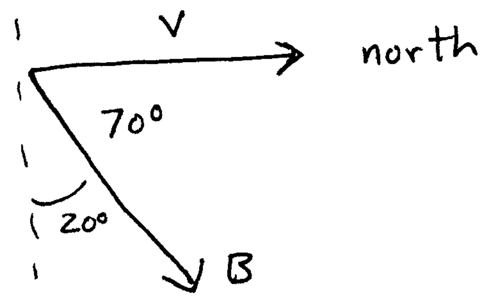
emf $= 0.025\,\text{V}$

By Lenz' Law and the RHR the increasing field must point upward to produce a clockwise emf.

$$\text{emf} = \frac{d}{dt} NBA = NA\frac{dB}{dt}$$

$$B \sim \frac{(\text{emf})(\Delta t)}{N \cancel{2}\pi a^2} = 0.01\,\text{T} \text{ out of page}$$

4.4



$$\text{emf} = \int_C (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$\vec{v} \times \vec{B} = vB\sin\theta \text{ west}$$

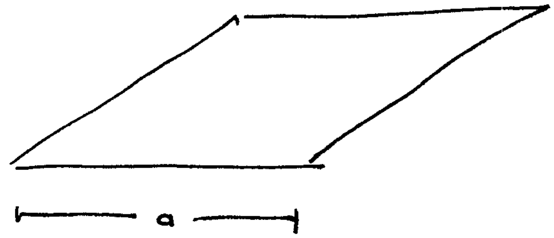
Let $d\vec{l}$ point west.

$$\text{emf} = vBl\sin\theta \text{ west}$$

$$= (300\,\text{m/s})(5\times10^{-5}\,\text{T})(20\,\text{m})\sin 70^\circ$$

$$= 0.28\,\text{V west}$$

4.6



$a = 1.25\text{cm}$

$B = 2.1\text{T}$

$R = 13\Omega$

$N = 137$

The flux goes from $\Phi^+ = \cancel{NB}$
$= NBa^2$

to $\Phi^- = -NBa^2$

The total ~~current~~ charge is the integral of the current

$$Q = \int I\,dt = \frac{1}{R}\int \text{emf}\,dt$$

by Ohm's Law

By the flux rule, $\text{emf} = -\dfrac{d\Phi}{dt}$

$$Q = \frac{1}{R}\int \text{emf}\,dt = -\frac{1}{R}\int \frac{d\Phi}{dt}\,dt = -\frac{1}{R}\int_{\Phi_+}^{\Phi_-} d\Phi$$

$$= -\frac{1}{R}\left(\Phi_- - \Phi_+\right) = \frac{2NBa^2}{R}$$

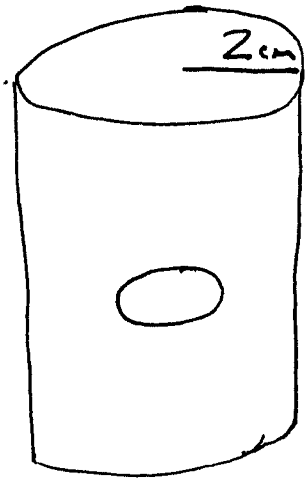
$$= \frac{2(137)(2.1\text{T})(0.0125\text{m})}{13\Omega} = 0.55\text{C}$$

$$[\Phi] = [NBA] = Vs$$

because $[\dot{\Phi}] = V$

$$Q = \frac{Vs}{\Omega} \checkmark \qquad A = \frac{V}{\Omega}$$

4.8



$N' = 1700\ \frac{1}{m}$

$$\frac{\Delta B}{\Delta t} = \cancel{\frac{1.2A}{0.4s} = 3}$$

$$\frac{\Delta B}{\Delta t} = \frac{N' \mu_0 I - 0}{\Delta t}$$

$$= \frac{(1700\ \frac{1}{m})(4\pi \times 10^{-7} \frac{Tm}{A})(1.2A)}{0.4s}$$

$$= 6.4 \times 10^{-3}\ \frac{T}{s}$$

$$\text{emf} = -\frac{\Delta \Phi}{\Delta t}$$

$$= NA \frac{\Delta B}{\Delta t}$$

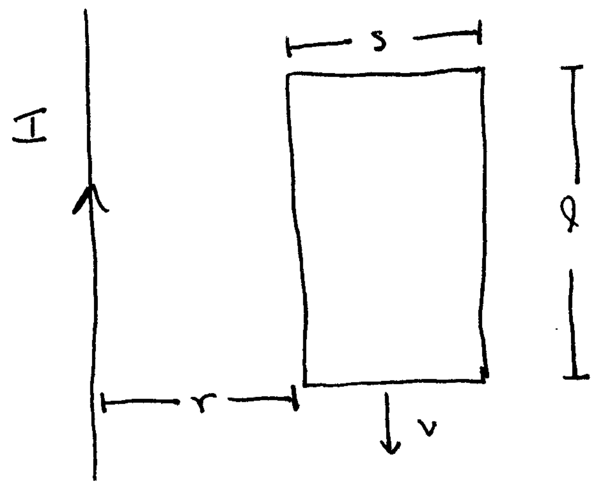
$$= A \frac{\Delta B}{\Delta t} \quad \text{since } N = 1$$

$$\text{emf} = \pi a^2 \frac{\Delta B}{\Delta t} = \pi (0.01m)^2\ 6.4 \times 10^{-3}\ T/s$$

$$= 2 \times 10^{-6}\ V$$

4.12 $I = 3\,A$, $r = 4\,cm$, $l = 5\,cm$

emf = $5.77 \times 10^{-8}\,V$ from solution



$s = 2.5\,cm$

$r = 4\,cm$

$l = 5\,cm$

The magnetic field is normal to the loop.

$$\Phi = l \int_r^{r+s} B(r)\,dr \qquad B(r) = \frac{\mu_0 I}{2\pi r}$$

$$= l \int_r^{r+s} \frac{\mu_0 I}{2\pi r}\,dr$$

$$= \frac{\mu_0 l I}{2\pi} \ln\left(\frac{r+s}{r}\right)$$

$$\cancel{= 4\pi \times 10}$$

$$\text{emf} = -\frac{d\Phi}{dt} = -\frac{\mu_0 I}{2\pi} \ln\left(\frac{r+s}{r}\right) \frac{dl}{dt}$$

$$\text{emf} = \frac{-\mu_0 I v}{2\pi} \ln\left(\frac{r+s}{r}\right)$$

$$v = \frac{2\pi \, \text{emf}}{\mu_0 I \ln\left(\frac{r+s}{r}\right)}$$

$$= \frac{2\pi (5.77 \times 10^{-8} V)}{\left(4\pi \times 10^{-7} \frac{Tm}{A}\right)(3A)\left(\ln\left(\frac{6.5}{4}\right)\right)}$$

$$= 0.2 m/s$$

4.14 $\quad \vec{E} = 17\hat{\phi}$

Faraday $\quad \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

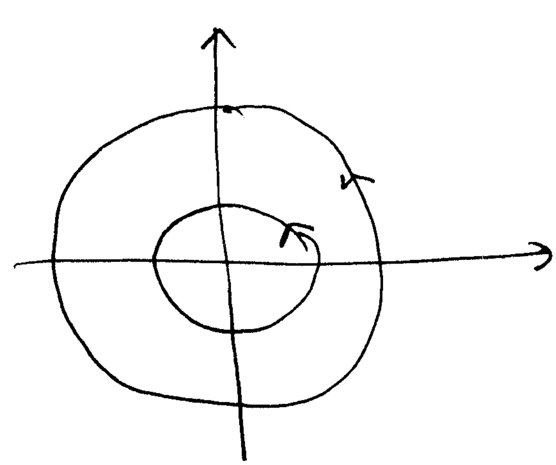
$$\nabla \times \vec{E} = \frac{1}{\rho}\frac{\partial(\rho B_\phi)}{\partial \rho}\,\hat{z}$$

$$= \frac{17}{\rho}\,\hat{z} = -\frac{\partial \vec{B}}{\partial t}$$

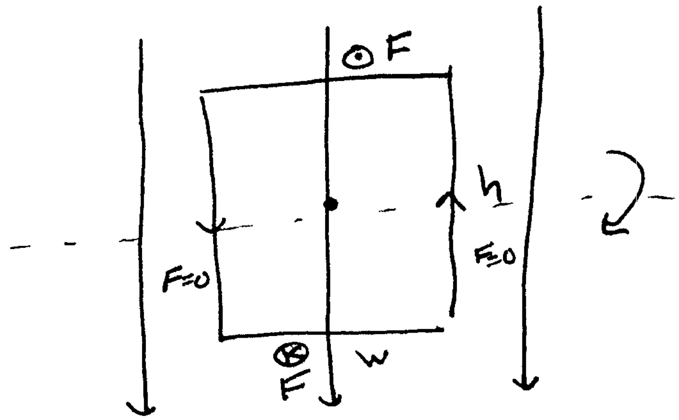
$$\vec{B} = -\frac{17t}{\rho}\,\hat{z} + \vec{B}_0$$

$\vec{B}$ is increasing in time and decreases as you move in the ρ direction.

$\vec{E}$



(4.16) $\vec{B} \perp \vec{a}$



Only top and bottom contribute to force.

$F_{top} = NwIB$ upward

$F_{bottom} = NwIB$ downward

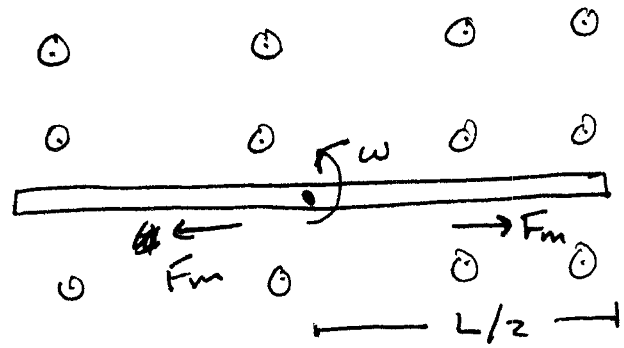
Torque $\vec{\tau} = \vec{\tau}_{top} + \vec{\tau}_{bottom} = \frac{h}{2} F_{top} + \frac{h}{2} F_{bottom}$ in the direction drawn

$|\vec{\tau}| = NhwIB = NaIB$

$= (17)(10 \times 10^{-4} m^2)(2A)(0.4T)$

$= 0.014 Nm$

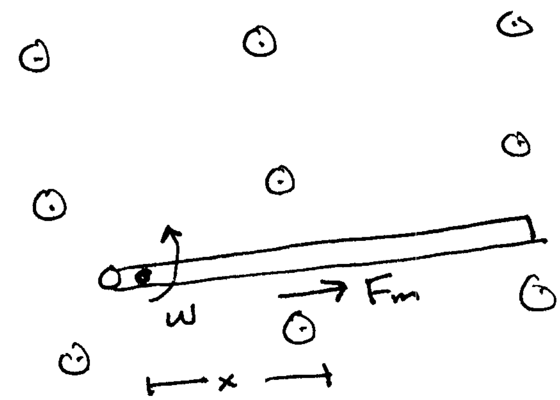
4.18



Let x be the distance from the center.

(a) The forces on the two ends cancel if the rod pivots about its middle, so emf = 0.

(b)



$$F_m = qvB = qx\omega B$$

$$\text{emf} = \frac{1}{q}\int_0^L F_m\,dx = \omega B \int_0^L x\,dx$$

$$= \frac{\omega B L^2}{2} = 68\,V$$