5.1

$$\vec{E} = E_0 \cos k(x-ct)\,\hat{y}$$

$$\vec{B} = B_0 \cos k(x-ct)\,\hat{z}$$

$\nabla\cdot\vec{E} = \nabla\cdot\vec{B} = 0$ by observation

<u>Ampere's Law</u>

$$\nabla\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & B_0\cos() \end{vmatrix}$$

$$= -\hat{y}\frac{\partial}{\partial x} B_0 \cos k(x-ct)$$

$$= +B_0 k \sin k(x-ct)\,\hat{y} \quad = \mu_0 \varepsilon_0 \frac{\partial\vec{E}}{\partial t}$$

$$= E_0 \mu_0 \varepsilon_0 k c \sin k(x-ct)\,\hat{y}$$

$$\Rightarrow B_0 k = \mu_0 \varepsilon_0 E_0 k c$$

$$\boxed{B_0 = \mu_0 \varepsilon_0 c E_0}$$ Condition 1

Faraday's Law

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_0 \cos(\,) & 0 \end{vmatrix}$$

$$= \hat{z}\,(-k)\,E_0 \sin k(x-ct)$$

$$= -\frac{\partial \vec{B}}{\partial t} = -B_0 k c \sin k(x-ct)\,\hat{z}$$

$$k E_0 = B_0 k c$$

$$\boxed{E_0 = B_0\, c} \qquad \text{Condition II}$$

Comparison with the previous condition shows

$$\sqrt{\mu_0 \varepsilon_0} = \frac{1}{c}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

If these conditions are met the given fields satisfy Maxwell's equations with $\rho = 0$, $\vec{J} = 0$.

(5.2) In this case,

$$\nabla\cdot\vec{B} \neq 0 \quad \text{so} \quad \cancel{\text{no}} \quad B_0 = 0.$$

$\nabla\cdot\vec{E} \neq 0$ so we have free charge lying around.

$\cancel{\nabla\times\vec{B}}$

The problem 5.1 says to assume $\Gamma = 0$, $\vec{J} = 0$ so we fail.

5.3 Check Maxwell's equations $\vec{E} = x\hat{y}$

$\nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$

Faraday

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & x & 0 \end{vmatrix} = \hat{z}$$

$$= -\frac{\partial \vec{B}}{\partial t}$$

$$\Longrightarrow \quad \vec{B} = -t\hat{z} + \vec{B}_0$$

$\nabla \times \vec{B} = 0$ ✓ Ampere Checks with $\vec{J} = 0$

So we need a time varying magnetic field.

5.4

5.4

$\vec{B} = x\,\hat{x}$

$\nabla \cdot \vec{B} = 1$ so this field will <u>never</u> be possible.

5.8

(a) $\vec{E} = \rho \hat{\rho}$ cylindrical implied

$\nabla \times \vec{E} = 0 \Rightarrow \vec{B} = 0$ or constant

$$\nabla \cdot \vec{E} = \frac{1}{\rho} \frac{\partial \rho A_\rho}{\partial \rho} = 2 \equiv \frac{\Gamma}{\varepsilon_0}$$

A uniform volume charge density $\Gamma = 2\varepsilon_0$

(b) $\vec{E} = \rho \hat{\phi}$

$\nabla \cdot \vec{E} = 0 \Rightarrow$ No net charge

$$\nabla \times \vec{E} = \frac{1}{\rho} \left(\frac{\partial \rho A_\phi}{\partial \rho} \right) \hat{z} = 2\hat{z} = -\frac{\partial \vec{B}}{\partial t}$$

$\vec{B} = -2t\hat{z} + \vec{B}_0$ Time varying field in $\hat{z}$ direction.

(c) $\vec{E} = \rho\,\hat{z}$

$$\nabla\cdot\vec{E} = 0$$

$$\nabla\times\vec{E} = -\frac{\partial A_z}{\partial \rho}\,\hat{\phi} = -\hat{\phi} = -\frac{\partial \vec{B}}{\partial t}$$

$\vec{B} = t\,\hat{\phi} + \vec{B}_0$ Circular time varying field.

$\cancel{\nabla\times\vec{B} = 0}$ $\qquad$ $\nabla\cdot\vec{B} = 0$

$$\nabla\times\vec{B} = \frac{t}{\rho}\,\hat{z} = \mu_0\,\vec{J}$$

$$\vec{J} = \frac{t}{\rho\,\mu_0}\,\hat{z}$$

5.6 (a) $\vec{B} = \rho \hat{\rho}$

$\nabla \cdot \vec{B} = 2$ Impossible

(b) $\vec{B} = \rho \hat{\phi}$

$\nabla \cdot \vec{B} = 0$ ✓

$$\nabla \times \vec{B} = \frac{1}{\rho}\left(\frac{\partial(\rho A_\phi)}{\partial \rho}\right)\hat{z} = 2\hat{z}$$

$$= \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

This field could be produced by

$$\vec{J} = \frac{2}{\mu_0}\hat{z} \quad \text{or} \quad \vec{E} = \frac{2t}{\mu_0 \varepsilon_0}\hat{z} + \vec{E}_0$$

(c) $\vec{B} = \rho \hat{z}$

$\nabla \cdot \vec{B} = 0$ ✓

$$\nabla \times \vec{B} = -\frac{\partial A_{\cancel{z}}}{\partial \rho}\hat{\phi} = -\hat{\phi} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

So try $\vec{J} = -\frac{1}{\mu_0}\hat{\phi}$, $\vec{E} = 0$ or

$$\vec{J} = 0, \qquad \vec{E} = -\frac{t}{\mu_0 \varepsilon_0}\hat{\phi} + E_0$$

Check Faraday

$$\nabla \times \vec{E} = \frac{1}{\rho}\left(\frac{\partial(\rho A_\phi)}{\partial \rho}\right)\hat{z}$$

$$= -\frac{t}{\mu_0 \varepsilon_0}\hat{z} = -\frac{\partial \vec{B}}{\partial t} \quad \text{Nope.}$$

So only $\vec{J} = -\frac{1}{\mu_0}\hat{\phi}$, $\vec{E} = 0$ works.

5.8 $\vec{J} = I_0 \frac{\hat{r}}{r^2}$

$$\nabla \cdot \vec{J} = I_0 \, \nabla \cdot \frac{\hat{r}}{r^2} = 4\pi I_0 \delta(\vec{r})$$

$$= \frac{\partial \rho}{\partial t}$$

$$I = \dot{Q} = \int_0^R \frac{\partial \rho}{\partial t} \, dv'$$

Current out of surface

Method I

$$= 4\pi I_0$$

Total Current Out

$$I = \int_S \vec{J} \cdot da = I_0 \int_S \frac{\hat{r}}{r^2} \cdot \hat{r} \, da$$

$$= \frac{I_0}{R^2} \int_S da$$

$$= \frac{I_0}{R^2} (4\pi R^2) = 4\pi I_0 \checkmark$$

5.12 Let the plates have radius R and let Q be the charge on one plate.

The charge density on the plate is

$$\sigma = \frac{Q}{\pi R^2}$$

and the field

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 \pi R^2}$$

Select an Amperian path with radius ~~R~~ ρ

The electric flux through the surface bounded by the path is

$$\Phi_e = EA = \pi \rho^2 E = \frac{\pi \rho^2 Q}{\varepsilon_0 \pi R^2}$$

$$= \frac{\rho^2 Q}{\varepsilon_0 R^2}$$

The displacement current is

$$I_d = \varepsilon_0 \frac{d\Phi_e}{dt} = \varepsilon_0 \frac{\rho^2}{\varepsilon_0 R^2} \frac{dQ}{dt}$$

$$= \frac{\varepsilon_0 \rho^2}{\varepsilon_0 R^2} I = \frac{\rho^2}{R^2} I$$

Apply Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = 2\pi\rho\, B(\rho) = \mu_0 I_d$$

$$= \frac{\mu_0 \rho^2}{R^2} I$$

$$B(\rho) = \frac{\mu_0 \rho}{2\pi R^2} I$$

Solutions to extra problems will be presented as part of review.