PHYS 3414 - Electricity and Magnetism- Homework Set 7

Chapter 8 and 9 - Conductors and Dielectrics

Due 5:00pm Wednesday March 26, 2008.

Good's Problems

8.2

8.6 20 pts

8.12

8.18 20 pts

8.22 20 pts

9.4

9.6

9.10 20 pts

9.14

9.16

9.24 20 pts

Notes, just compute power 8.2

8.~~18~~8, one of the parts has infinite resistance.

9.16 Dielectric from $r_1 = 0.2cm$ $r_d = 0.45cm$

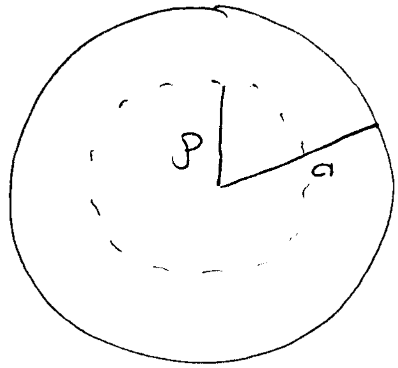
8.2 Conductivity of copper $\sigma = 5.8 \times 10^{7}\ S/m$

$$J = \sigma E = 5.8 \times 10^{9}\ \frac{A}{m^2}$$

$$\frac{dP}{dv} = \vec{J} \cdot \vec{E} = 5.8 \times 10^{11}\ \frac{W}{m^3}$$

which is huge.

8.6



Thickness s

$$\text{emf} = 2\pi\rho E = -\dot{\Phi} = -\pi\rho^2 B_0 \, d\cos\omega t \quad \text{(Faraday}$$

$$= \pi\rho^2 B_0 \,\omega \sin\omega t$$

$$E = \frac{\rho B_0 \omega}{2} \sin\omega t$$

$$J = \frac{\sigma\rho B_0 \omega}{2} \sin\omega t$$

$$\frac{dP}{dv} = \vec{E}\cdot\vec{J} = \rho^2\sigma\left(\frac{B_0\omega}{2}\right)^2 \sin^2\omega t$$

$$P = \int\left(\frac{dP}{dv}\right)dv' = s\int_0^{2\pi} d\phi' \int_0^a \rho' d\rho' \, \frac{dP}{dv}$$

$$P = 2\pi s\, \sigma \sin^2\omega t \left(\frac{B_0\,\omega}{2}\right)^2 \int_0^a \rho'^3\, d\rho'$$

$$= 2\pi s\, \sigma \sin^2\omega t \left(\frac{B_0\,\omega}{2}\right)^2 \frac{a^4}{4}$$

$$= \frac{\left(\pi a^4 s\right)}{8}\, \sigma B_0^2 \omega^2 \sin^2\omega t$$

(8.12) The charge on the sphere is found from the voltage $V = \frac{kQ}{R}$

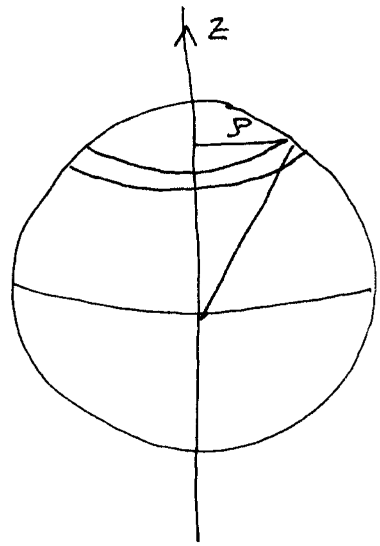
$$Q = \frac{VR}{k}$$

$$\sigma = \frac{Q}{\text{Area}} = \frac{VR}{4\pi R^2 k}$$

$$= \frac{V}{4\pi k R} = \frac{\varepsilon_0 V}{R}$$

The magnetic field of a current loop along the axis is

$$|\vec{B}(z)| = \frac{\mu_0 I a^2}{2(z^2+a^2)^{3/2}} \quad (\text{Good } 3.9)$$



Cut the surface of the sphere up into strips dI

$$dI = \underbrace{(\sigma R d\theta)}_{\lambda}\underbrace{\omega \rho}_{v}$$

$$\rho = R \sin\theta \qquad z = R\cos\theta$$

$$dI = \sigma R^2 \omega \cancel{\rho} \sin\theta \, d\theta$$

$$\cancel{d\vec{B}(\theta) = \mu_0 \int}$$

$$dB(\theta) = \frac{\mu_0 \, dI \, \rho'^2}{2((z-z')^2 + \rho'^2)^{3/2}}$$

~~we~~ We want the field at the origin so $z = 0$.

$$dB(\theta) = \frac{\mu_0 \, (R\sin\theta)^2 \, dI}{2\left((R\cos\theta)^2 + (R\sin\theta)^2\right)^{3/2}}$$

$$= \frac{\mu_0}{2R} \sin^2\theta \, dI$$

$$= \frac{\mu_0}{2R} \sigma R^2 \omega \sin^3\theta \, d\theta$$

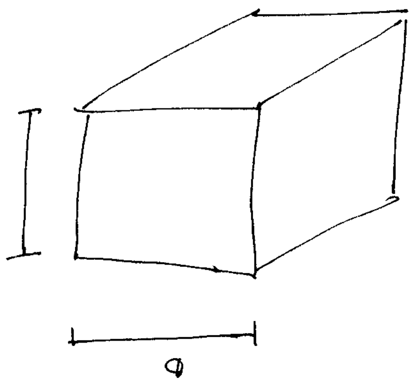
$$B = \int_0^{\pi} \frac{\mu_0 R \omega \sigma}{2} \sin^3\theta \, d\theta \qquad \int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3}$$

$$= \frac{4}{3} \frac{\mu_0 R \omega}{2} = \frac{2}{3} \mu_0 R \omega \sigma$$

Substitute $\sigma = \varepsilon_0 V / R$

$$B = \frac{2}{3} \varepsilon_0 \mu_0 V \omega = 7.8 \times 10^{-11} T \text{ (not much)}$$

8.18



$\sigma = \cancel{\sigma_0 \left(1 - \frac{x}{}\right)}$

$\sigma_0 \dfrac{x}{a}$

The current density must have the same magnitude at all point, $J_0 \hat{x}$.

By Ohm's law, $\vec{J} = \sigma \vec{E}$ or

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{a J_0}{\sigma_0 x} \hat{x}$$

$$\Delta V = -\int_0^a \frac{a J_0}{\sigma_0 x}\, dx \rightarrow \infty$$

$$R = \frac{\Delta V}{I} \rightarrow \infty$$

~~(b)~~ $\cancel{\sigma = \sigma_0 \left(\frac{x}{a}\right)^2}$ $\cancel{\vec{E} = \frac{\vec{J}}{\sigma} = \frac{\sigma_0 a^2 J_0}{x^2}}$

(b) $\sigma = \sigma_0 \sqrt{\frac{x}{a}}$

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{\sqrt{a}\, J_0}{\sigma_0\, x^{1/2}}$$

$$\Delta V = -\int_0^a E\, dx = -\frac{\sqrt{a}\, J_0}{\sigma_0} \int_0^a x^{-1/2}\, dx$$

$$= -\frac{\sqrt{a}\, J_0}{\sigma_0} \left. \frac{x^{1/2}}{1/2} \right|_0^a$$

$$= \frac{-2 a J_0}{\sigma_0}$$

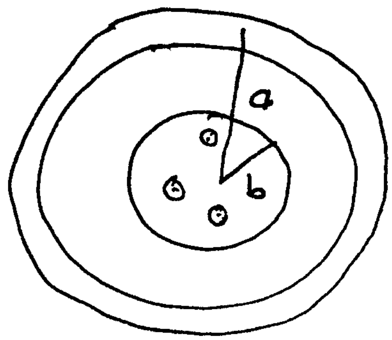
$$R = \frac{\Delta V}{I} = \frac{2 a J_0 / \sigma_0}{a^2 J_0} \qquad I = a^2 J_0$$

$$= \frac{2}{\sigma_0 a}$$

8.22

$a = 1.5\,cm$ $\quad b = 1.2\,cm$ $\quad A = 0.5\,cm^2$

$\sigma = 5.8\times10^{-7}\,S/m$



Resistance of Ring

$$R = \frac{l}{\sigma A} = \frac{2\pi a}{\sigma A} = \frac{2\pi(1.5\times10^{-2}m)}{(5.8\times10^{7}S/m)(0.5\times10^{-4}m^2)}$$

$$= 3.2\times10^{-5}\,\Omega$$

Magnetic Flux

$$\Phi_m = B\pi b^2$$

$B = B_0 \cos\omega t$

$B_0 = 0.2T$

$\omega = 377\,Hz$

Faraday

$$emf = -\dot{\Phi}_m = B_0\omega\pi b^2 \sin\omega t$$

Ohm's Law

$$I(t) = \frac{emf}{R} = \frac{B_0 \omega \pi b^2}{R} \sin\omega t$$

$$= (1060\ A) \sin\omega t$$

9.4

$$\vec{p} = \frac{4}{3}\pi R^3 \vec{P}$$

9.6

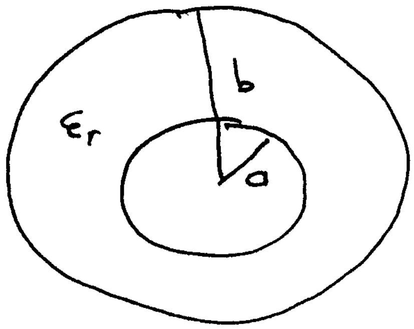
$$\varepsilon_r = 1000$$

$$C = \varepsilon_r C_0 = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

$$= \frac{(1000)\left(8.85\times10^{-12}\,\frac{C^2}{Nm^2}\right)(\pi)(0.02m)^2}{0.02m}$$

$$= 560\,pF$$

9.10



Add $+Q$ to inner conductor

For $a<r<b$, $\quad \vec{E} = \dfrac{kQ}{\varepsilon_r r^2}\hat{r}$

The potential difference is

$$\Delta V = -\int_a^b E\,dr = \frac{kQ}{\varepsilon_r}\frac{1}{r}\Big|_a^b$$

$$= \frac{kQ}{\varepsilon_r}\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$|\Delta V| = \frac{kQ}{\varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{Q}{|\Delta V|} = \frac{4\pi\varepsilon_0\varepsilon_r}{\left(\frac{1}{a} - \frac{1}{b}\right)} = 5.1\,pF$$

9.14 $\quad \varepsilon_r = \varepsilon_1 + (\varepsilon_2 - \varepsilon_1)\frac{x}{s}$

There is only free charge on the plates, so

D is constant $\quad \vec{D} = \sigma \hat{z}$ from class

$$\vec{D} = \varepsilon \vec{E} = \varepsilon_r \varepsilon_0 \vec{E}$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0\left(\varepsilon_1 + (\varepsilon_2 - \varepsilon_1)^{x}/_{s}\right)}$$

$$\Delta V = -\int_0^s E\,dx = -\frac{\sigma}{\varepsilon_0}\int \frac{dx}{\varepsilon_1 + (\varepsilon_2 - \varepsilon_1)^{x}/_{s}}$$

$$= -\frac{\sigma s}{\varepsilon_0}\left[\frac{\ln\left(\varepsilon_1 + (\varepsilon_2 - \varepsilon_1)^{x}/_{s}\right)}{\varepsilon_2 - \varepsilon_1}\right]_0^s$$

$$= \frac{\sigma s}{\varepsilon_0(\varepsilon_2 - \varepsilon_1)} \ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right)$$

$$C = \frac{\sigma A}{|\Delta V|} = \frac{\varepsilon_0(\varepsilon_2 - \varepsilon_1)A}{s \ln\left(^{\varepsilon_2}/_{\varepsilon_1}\right)}$$

9.16

$a = 0.2\,\text{cm}$ $\qquad$ $b = 6.7\,\text{cm}$

$\varepsilon_r = 2.3$

Dielectric from a to c $\qquad$ $c = \dfrac{a+b}{2} = \cancel{4.5\,\text{cm}}$ $0.45\,\text{cm}$

From lecture, capacitance per unit length

$$C = \frac{2\pi\varepsilon_0}{\ln(r_2/r_1)}$$

Model system as two capacitors in series.

$$\underbrace{C_1 = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(c/a)}}_{\text{dielectric-filled}} \qquad C_2 = \frac{2\pi\varepsilon_0}{\ln(b/c)}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{1}{\frac{\ln(c/a)}{2\pi\varepsilon_0\varepsilon_r} + \frac{\ln(b/c)}{2\pi\varepsilon_0}}$$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{c}{a}\right) + \varepsilon_r \ln\left(b/c\right)}$$

$$= 70\,pF$$

9.24

The force in capacitor exerts on the fluid

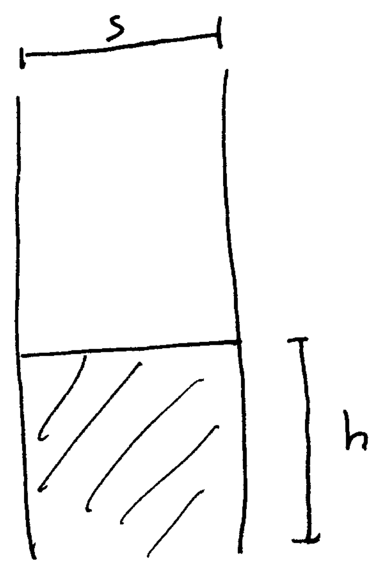
is

$$F = \frac{W^2 w \epsilon_0}{2s} (\epsilon_r - 1) \qquad \text{(Good 9.41)}$$

$w =$ width of the plates (square)

$s =$ separation

To support the fluid, this force must equal the weight of the column raised.



The volume of fluid is $Vol = s h w$

$$F = mg = \rho \cdot Vol \cdot g = \rho s h w g$$

where now ρ is the mass density.

$$F = \frac{V^2 w \varepsilon_0}{2s} (\varepsilon_r - 1) = \rho s h w g$$

$$h = \frac{V^2 \varepsilon_0 (\varepsilon_r - 1)}{2 \rho s^2 g} = 0.45\,\text{mm}$$