PHYS 3414 - Electricity and Magnetism- Homework Set 8

Chapter 10 - Magnetic Materials

Due 5:00pm Wednesday April 2, 2008.

Good's Problems

10.4 20

10.6 (Don't do $\vec{A}$) 20

10.8 20

10.10 20

10.14 20

10.16 ($V_1 = 120$V)

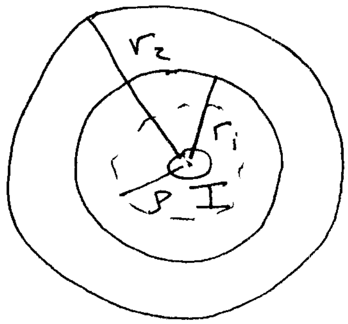
10.20

10.31

10.4

$$m = MV = \frac{4}{3}\pi R^3 M$$

(10.6)



The free current is I.

$$\nabla \times \vec{H} = J_f$$

$$\oint \vec{H} \cdot d\vec{l} = \mu_0 I_{f,enc}$$

By symmetry, the field is circular.

$$\oint \vec{H} \cdot d\vec{l} = 2\pi\rho H = \cancel{\mu_0} I$$

$$\vec{H} = \cancel{\mu_0} \frac{I}{2\pi\rho} \hat{\phi}$$

For $r < r_1$ and $r > r_2$, $M = 0$

$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi\rho} \hat{\phi}$$

Inside iron $(r_1 < r < r_2)$

$$\vec{B}_i = \mu_0 \mu_r \vec{H}$$

$$\vec{B}_i = \frac{\mu_0 \mu_r I}{2\pi\rho} \hat{\phi}$$

Surface Currents

Inner surface $\hat{n} = -\hat{\rho}$

$$\vec{K} = \vec{M} \times \hat{n}$$

$$\vec{M} = \chi_m \vec{H} = \frac{\chi_m I}{2\pi\rho} \hat{\phi} = \frac{\chi_m I}{2\pi r_1} \hat{\phi}$$

$$\vec{K}_1 = \vec{M} \times \hat{n} = \frac{\chi_m I}{2\pi r_1} \hat{\phi} \times (-\hat{\rho})$$

$$= \frac{\chi_m I}{2\pi r_1} (-\hat{z})$$

Likewise for the outer surface

$$\vec{K}_2 = \frac{\chi_m I}{2\pi r_2} \hat{z}$$

Total Current $\quad I_1 = 2\pi r_1 K_1 = -\chi_m I$

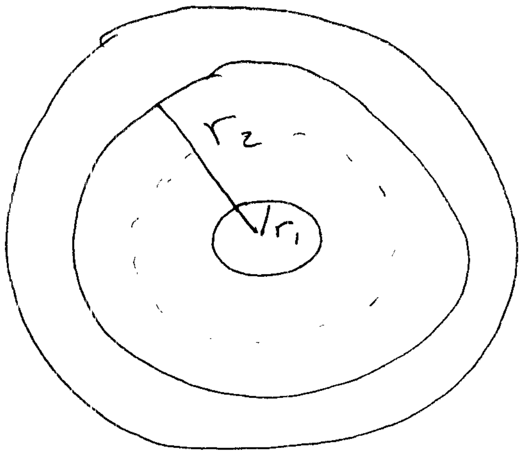
$$I_2 = 2\pi r_2 K_2 = \chi_m I$$

<u>Alternate Methods</u>

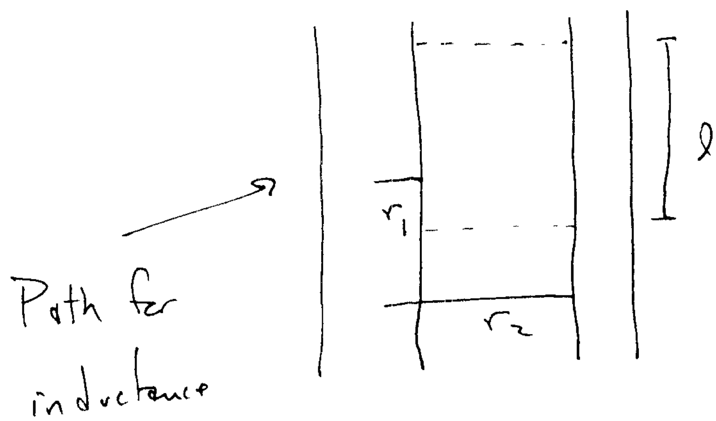
(1) Use Stokesian loop around surface

(2) Use $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

10.8



Side Cut Away



The magnetic field (from memory) is

$$\vec{B} = \frac{\mu_0 I}{2\pi\rho}\,\hat{\phi}$$

if I flows through the inner conductor.

The flux through a square surface of length l between the conductors is

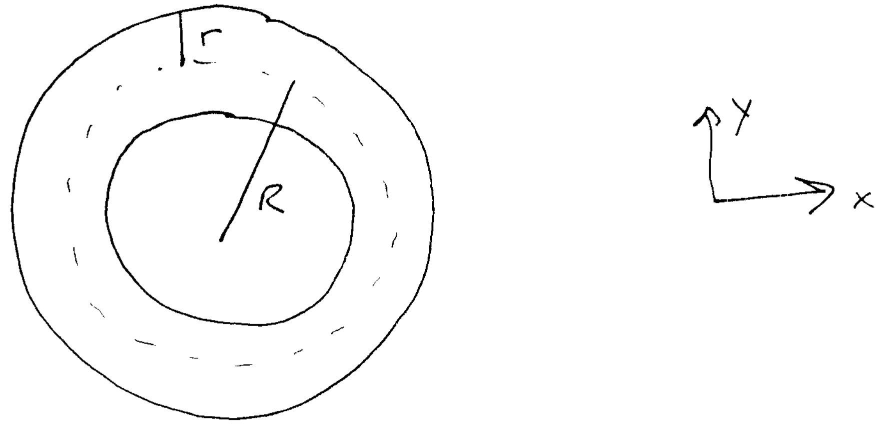
$$\Phi = \int \vec{B}\cdot d\vec{a} = l\int_{r_1}^{r_2} \frac{\mu_0 I}{2\pi\rho}\,d\rho$$

$$= \cancel{2}\,\frac{\mu_0 I l}{2\pi}\ln\left(\frac{r_2}{r_1}\right)$$

Self-inductance

$$L = \frac{\Phi}{I} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

10.10



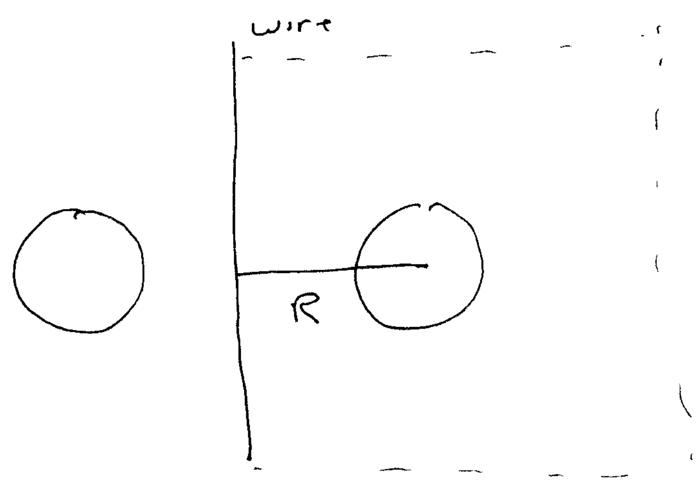
The magnetic field is circular. The current flowing through the surface bounded by an Amperian path of radius $R-r < \rho < r+R$ is NI

Ampere's law gives

$$\oint \vec{B}\cdot d\vec{l} = 2\pi\rho B = \mu_0 N I$$

$$\vec{B} = \frac{\mu_0 N I}{2\pi\rho}\hat{\phi}$$

Side View



surface flux is computed through

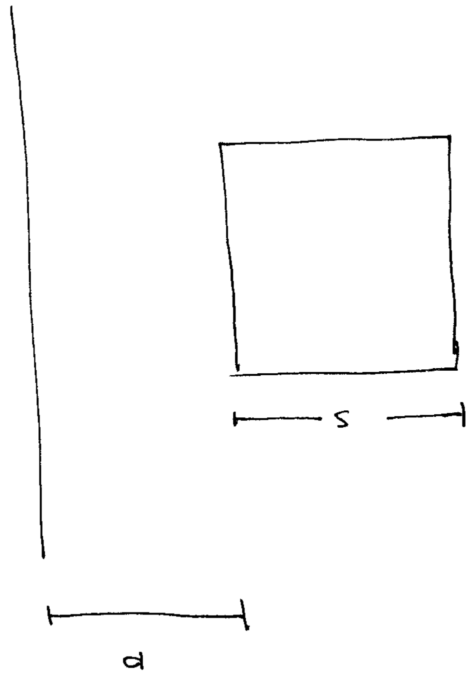
Assuming flux is uniform over toriod,

$$\Phi = \cancel{\pi B} \; \pi r^2 B(R)$$

$$= \frac{\pi r^2 \mu_0 N I}{2\pi R}$$

$$M = \frac{\Phi}{I} = \frac{\pi r^2 \mu_0 N}{2\pi R}$$

10.14



Field of wire $\quad B = \dfrac{\mu_0 I}{2\pi\rho}$

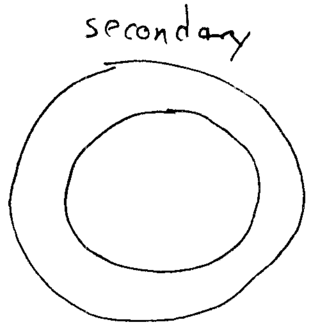
Flux through loop

$$\Phi = s \int_a^{a+s} \frac{\mu_0 I}{2\pi\rho}\, d\rho$$

$$= \frac{s \mu_0 I}{2\pi} \ln\left(\frac{a+s}{s}\right)$$

$$M = \frac{\Phi}{I} = \frac{s\mu_0}{2\pi} \ln\left(\frac{a+s}{s}\right) = 8.3\times10^{-9}\,H$$

10.16



$l = 1.0\,cm$

$A = 1.3\,mm \times 3.0\,mm$

The resistance of the secondary is

$$R = \frac{l}{\sigma A} = \frac{0.01\,m}{(1.0\times10^{7}\,S/m)(0.0013\,m)(0.003\,m)}$$

$$= 2.56\times10^{-4}\,\Omega$$

The power is conserved between the primary and secondary

$$I_1 V_1 = I_2 V_2$$

$$\cancel{P_2 = \frac{V_2^2}{R} \qquad V_2 = \sqrt{P_2 R}}$$

$$\cancel{= \sqrt{100.}}$$

$$P_2 = 100\,W = I^2 R$$

$$I_2 = \sqrt{\frac{100\,W}{R_2}} = 625A$$

$$V_2 = \frac{P_2}{I_2} = 0.16\,V$$

$$P_1 = 100\,W = I_1 V_1$$

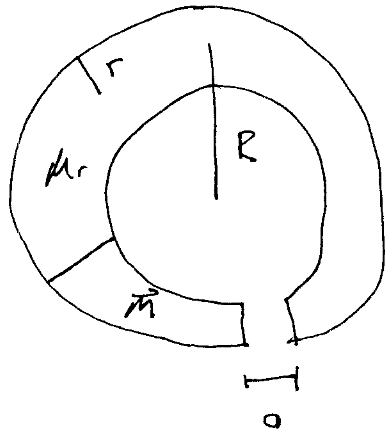
$$I_1 = \frac{100\,W}{V_1} = \frac{100\,W}{120\,V} = 0.83\,A$$

Turns

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad N_1 = \frac{V_1}{V_2} N_2 \qquad N_2 = 1$$

$$= \frac{120\,V}{0.16\,V} = 750 \text{ turns}$$

10.20



$R = 8.1\text{cm}$

$r = 0.72\text{cm}$

$\mu_r = 100$

$M = 10^5 \text{ A/m}$

$a = 0.7\text{cm}$

$$\text{mmf} = \oint \vec{H} \cdot d\vec{l} = 0$$

H_m = Field in magnet

H_i = Field in iron - Parallel to

H_o = Field in gap - Parallel to B

Using a Gaussian pillbox, $\nabla \cdot \vec{B} = 0 \Rightarrow$ B is continuous around the circle

$$2\pi R\left(\frac{9}{10}\right) H_i + a H_o = 2\pi R \frac{1}{10} H_m$$

H_m is opposite direction to B in permanent magnet.

$$2\pi R \left(\frac{9}{10}\right) H_i + a H_o = 2\pi R \left(\frac{1}{10}\right) H_m$$

$$H_m = 9 H_i + \frac{10a}{2\pi R} H_o$$

$$B = H_o / \mu_o$$

$$B = H_i / \mu_o \mu_r$$

$$= 9 \frac{B}{\mu_o \mu_r} + \frac{10a}{2\pi R} \frac{B}{\mu_o}$$

$$\mu_o H_m = \left(\frac{9}{\mu_r} + \frac{10a}{2\pi R}\right) B$$

In the permanent magnet, B and H_m point in opposite directions.

$$B = \mu_o H_m + \mu_o M = -\left(\frac{9}{\mu_r} + \frac{10a}{2\pi R}\right) B + \mu_o M$$

$$B \left(1 + \frac{9}{\mu_r} + \frac{10a}{2\pi R}\right) = \mu_o M$$

$$B = \frac{\mu_o M}{\left(1 + \frac{9}{\mu_r} + \frac{10a}{2\pi R}\right)} = 0.102\,T$$