Lecture 11 - Ampere's Law

Ampere's Law

$$\nabla \times \vec{B} = \mu_0 (\vec{J} + \vec{J}_d)$$

$$J_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

-or-

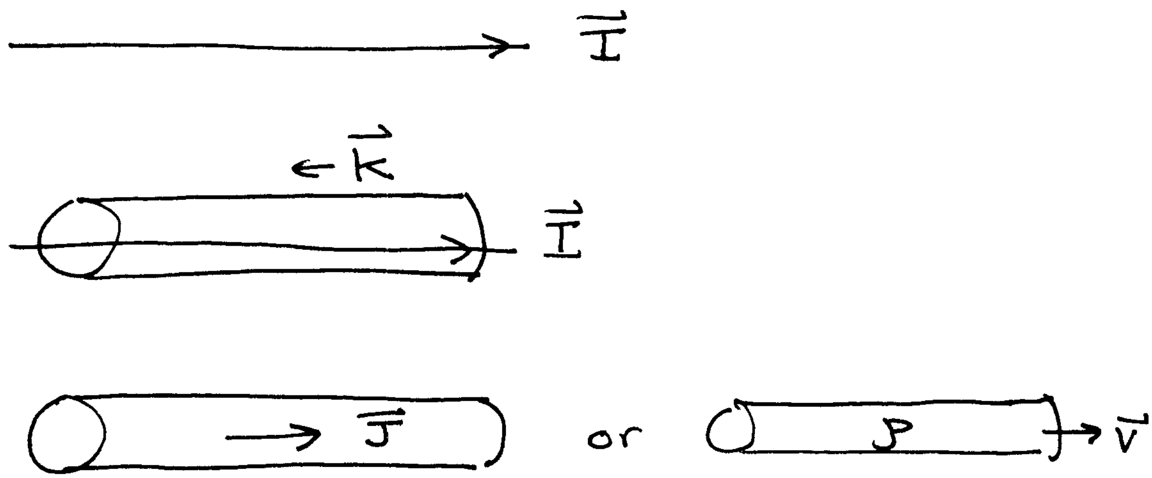
$$\oint_c \vec{B} \cdot d\vec{l} = \mu_0 (I + I_d)$$

First, both set of equations are completely true, so we can now add on extra test to whether the combination $\vec{E}, \vec{B}$ is a possible electric-magnetic field combination.

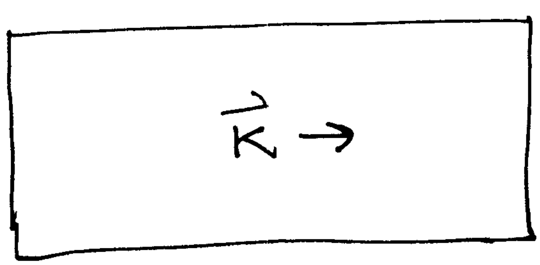
The integral form is extremely powerful if the system has enough symmetry. It doesn't matter if the source of the current is a flow of particles or a changing electric field. So for any of the systems that follow we could use $\vec{I}$ or $\vec{I}_d$, $\vec{K}$ or $\vec{K}_d$, or $\vec{J}$ or $\vec{J}_d$.

Systems

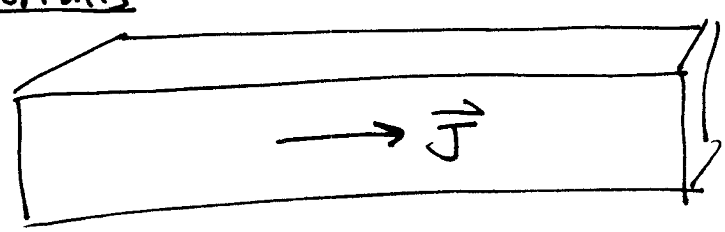
Cylindrical Wires



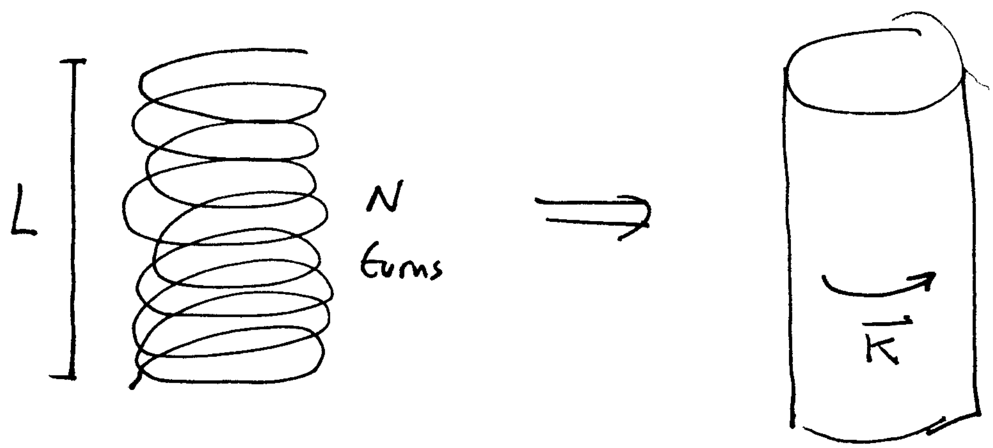
Sheet Currents



Volume Currents

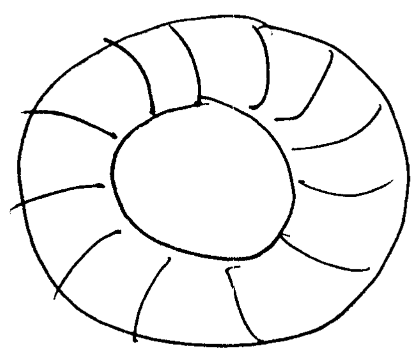


Solenoids



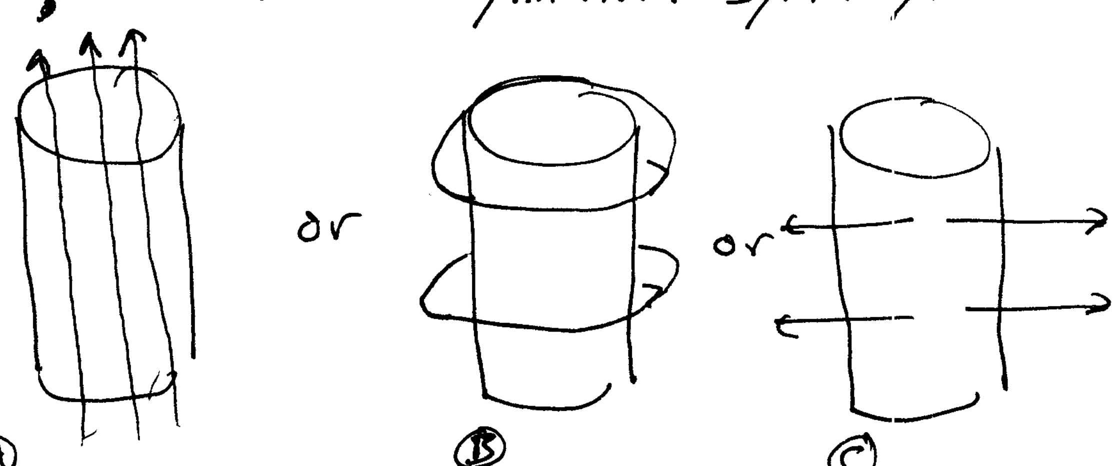
$n = N' = N/L$ $\qquad$ $\vec{K} = N'I$

Toriod



Infinite Solenoid

What does the field look like, It must have cylindrical symmetry.



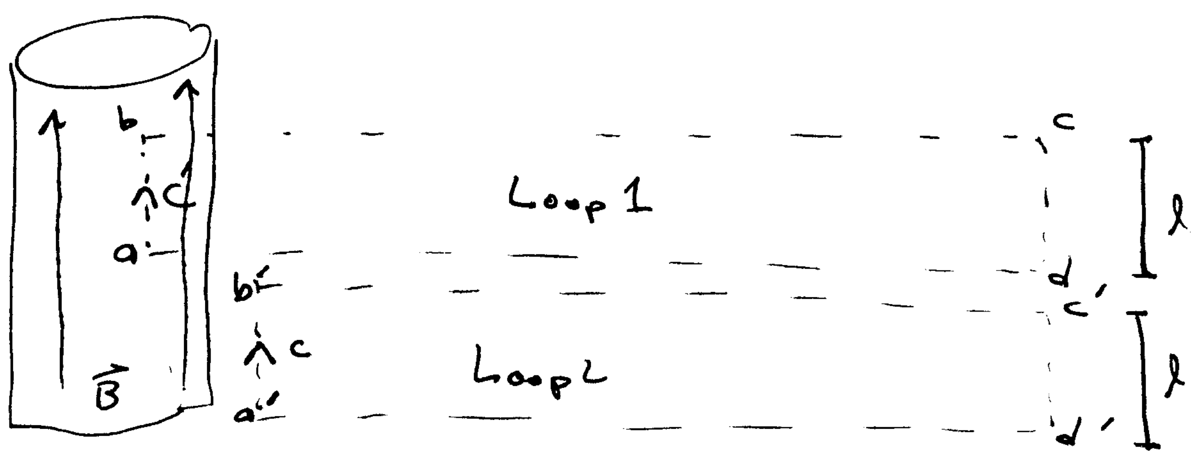
Essentially, A, B, and C are the $\hat{\rho}, \hat{\phi}, \hat{z}$ components in cylindrical coordinates.

(A) is not going to work because the radial field line must start somewhere, $\Rightarrow \nabla \cdot \vec{B} \neq 0$.

(B) Ampere's Law implies this field is zero since $I_{enc} = 0$

(C) So the field is in the z-direction.

Far from the solenoid, by the Biot-Savart law the opposite direction currents cancel, so $B \to 0$ as $\rho \to \infty$.



Since $\vec{B} \perp$ bc, da, b'c', d'a', those segment contribute zero to $\oint_C \vec{B} \cdot d\vec{l}$

Since B is zero at ∞, cd and c'd' do not contribute.

For loop 1,

$$\oint \vec{B} \cdot d\vec{l} = Bl = \mu_0 I_{enc} = \mu_0 N' l I$$

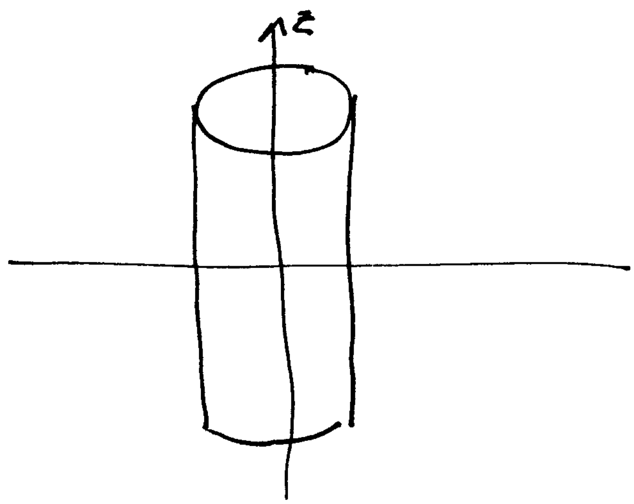
$$B_{inside} = \mu_0 N' I = \mu_0 K$$

For loop 2, $\oint \vec{B} \cdot d\vec{l} = Bl = 0$

$$B_{outside} = 0$$

Now consider a cylindrical region $\rho < a$ that contains an electric field

$$\vec{E} = E_0 e^{-t/\tau} \hat{z}$$



Compute the charge density, current density, and magnetic field.

Charge density $\quad \varepsilon_0 \nabla \cdot \vec{E} = \rho = 0$

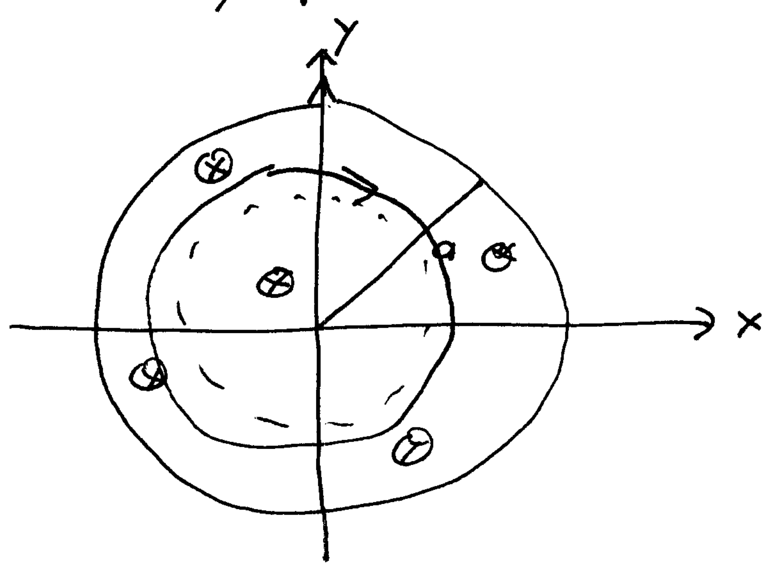
Current density - No charge so without additional info, no current $\vec{J} = 0$

Displacement current

$$\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$= -\frac{\varepsilon_0}{\tau} e^{-t/\tau} \hat{z}$$

<u>Work in the x-y plane</u>



E points upward, $\vec{J}_d$ downward.

What shape is the field?

No $\hat{\rho}$ as before.

Since this is a bundle of line currents, it has no z-component.

$\Rightarrow$ Field is circular, $\hat{\phi}$.

By RHR, $\vec{B}$ clockwise $(-\hat{\phi})$

For $\rho < a$, $I_{enc} = \int_0^{\rho} d\rho\, \rho \int_0^{2\pi} d\phi\; J_d(\rho, \phi)$

$$= -\frac{\varepsilon_0 E_0}{\tau} \int_0^{\rho} d\rho\, \rho \int_0^{2\pi} d\phi\; e^{-t/\tau}$$

$$= -\pi \rho^2 \frac{\varepsilon_0 E_0}{\tau} e^{-t/\tau}$$

$$\oint_c \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} = -\mu_0 \pi \rho^2 \frac{\varepsilon_0 E_0}{\tau} e^{-t/\tau}$$

$$\vec{B} = B(\rho)\,\hat{\phi} \qquad d\vec{l} = \rho\, d\phi\, \hat{\phi}$$

$$\oint_c \vec{B} \cdot d\vec{l} = B(\rho)\,\rho \int_0^{2\pi} d\phi = 2\pi\rho\, B(\rho)$$

$$2\pi\rho B(\rho) = -\pi\rho^2 \frac{E_0 \varepsilon_0}{\tau} e^{-t/\tau} \mu_0$$

$$\vec{B}(\rho) = -\frac{E_0 \varepsilon_0 \mu_0}{2\tau} \rho e^{-t/\tau} \hat{\phi}$$

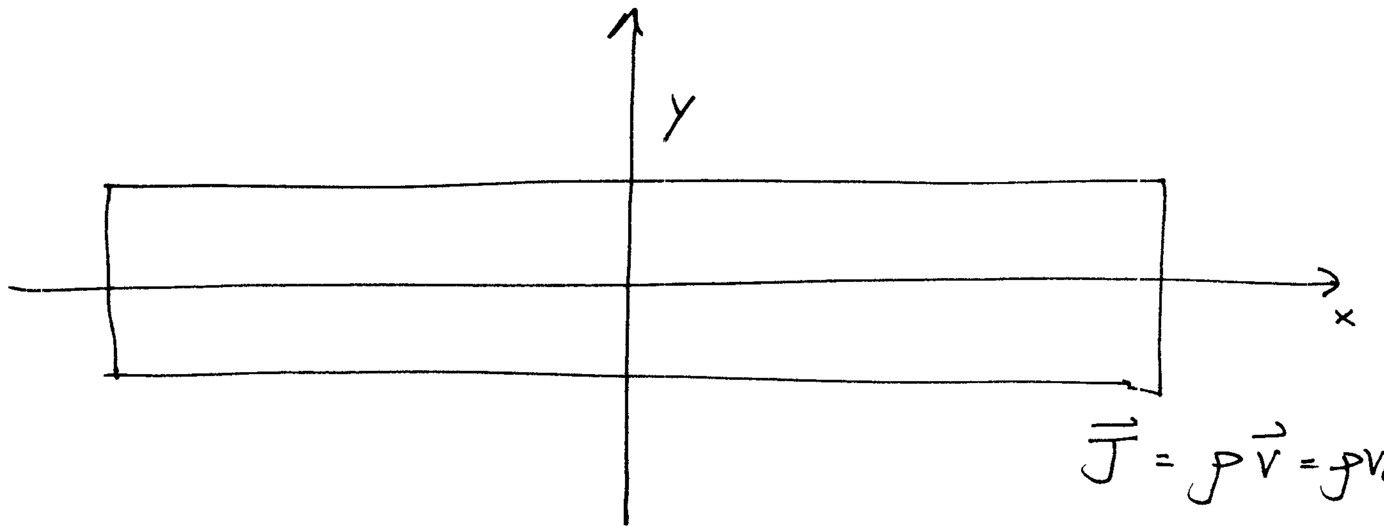
For $\rho > a$,

$$I_{enc} = -\pi a^2 \frac{E_0 \varepsilon_0}{\tau} e^{-t/\tau} \mu_0$$

$$\vec{B}(\rho) = -a^2 \frac{E_0 \varepsilon_0 \mu_0}{2\tau\rho} e^{-t/\tau} \hat{\phi}$$

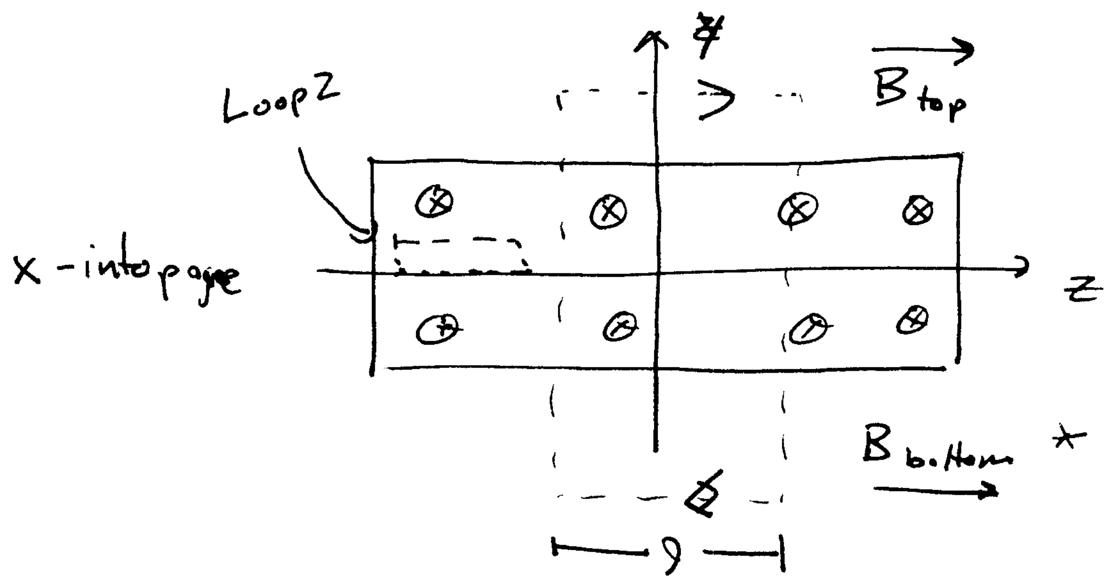
Problems – This field is not causal. Must have left stuff out.

Ex Slab of charge density ρ moving in the x-direction with velocity $\vec{v} = v_0 \hat{x}$. Compute $\vec{B}$.



$\vec{J} = \rho \vec{v} = \rho v_0 \hat{x}$

<u>End view</u>



B_{bottom} will end up pointing $-\hat{x}$

$$I_{enc} = 2aJl = 2a\rho v_0 l$$

$$\oint \vec{B}\cdot d\vec{l} = B_{top}\, l - B_{bottom}\, l = \mu_0 I_{enc} = 2a\rho v_0 \mu_0 l$$

By the rotation symmetry in the Biot-Savart law,

$$B_{top} = -B_{bottom}$$

$$2B_{top}\,l = \mu_0 2a\rho v_0 l$$

$$\vec{B}_{top} = \mu_0 a\rho v_0 \hat{z} = -\vec{B}_{bottom}$$

$$\vec{B}(y=0) = 0$$

<u>Loop 2</u>

$$B(y)\,l - 0 = \mu_0 (\rho v_0 \cancel{z} l)$$

$$\vec{B}(y) = \mu_0 v_0 y \rho \hat{z}$$