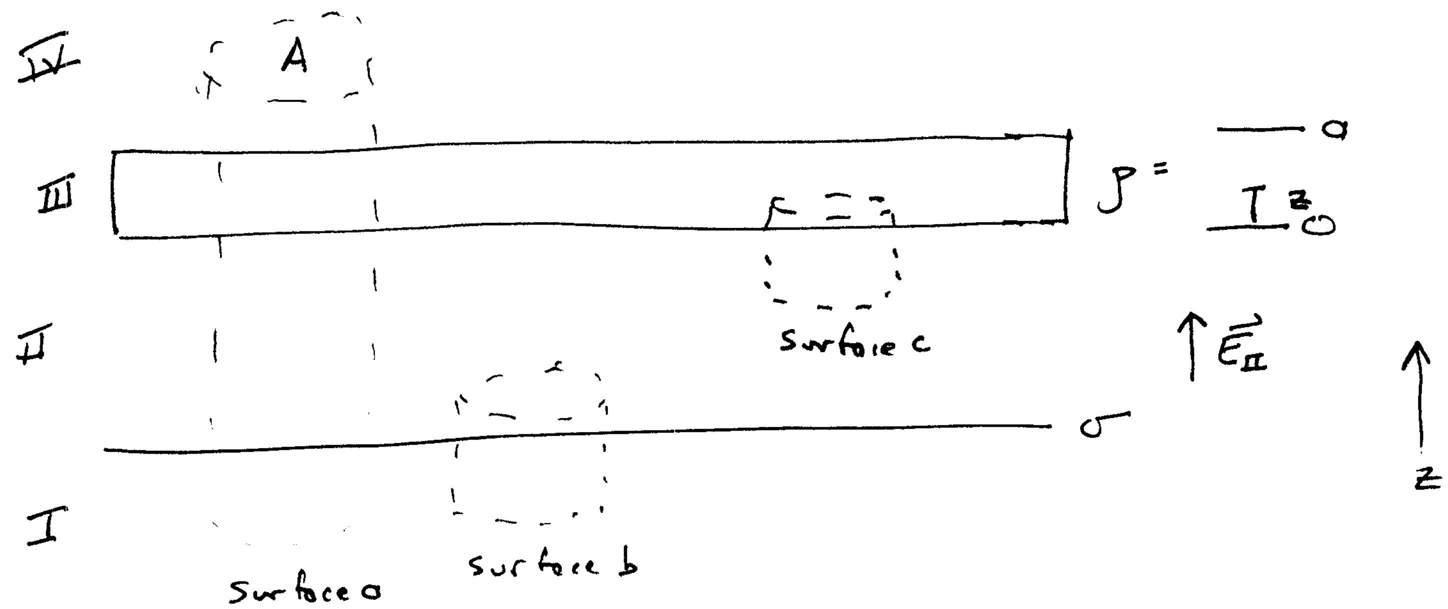
- Lecture 8 - Electric Pressure II



Let $\rho a = -\sigma$

Solve for the field using Gauss law

<u>Surface a</u> $\quad Q_{enc} = Aa\rho + \sigma A = -\sigma A + \sigma A = 0$

$$\vec{E}_{I} = \vec{E}_{IV} = 0$$

<u>Surface b</u> $\quad Q_{enc} = \sigma A$

$$\underline{\Phi} = E_{II} A = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$\vec{E}_{II} = \frac{\sigma}{\varepsilon_0}\hat{z}$$

<u>Surface c</u> $\quad Q_{enc} = A\cancel{z}\rho$

$$\underline{\Phi} = E_{III} A + (-E_{II} A) = \frac{Q_{enc}}{\varepsilon_0} = \frac{Az\rho}{\varepsilon_0}$$

$$E_{III} = E_{II} + \frac{z\rho}{\varepsilon_0}$$

$$= \frac{\sigma}{\varepsilon_0} + \frac{z}{\varepsilon_0}\left(\frac{-\sigma}{a}\right)$$

$$= \frac{\sigma}{\varepsilon_0}\left(1 - \frac{z}{a}\right)$$

Compute the force exerted on top slab (let it be square $l \times l$)

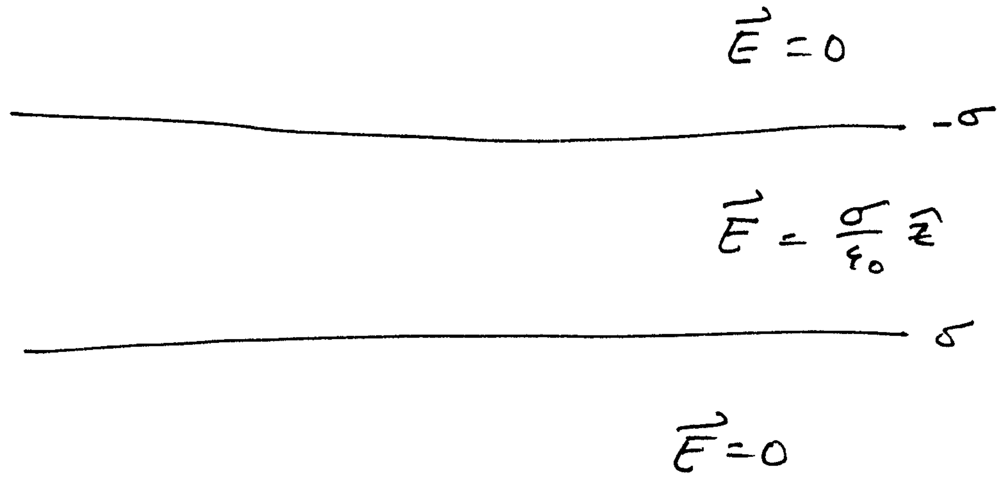
$$|\vec{F}| = \int_0^l dx \int_0^l dy \int_0^a dz \; \rho \vec{E}$$

$$= l^2 \rho \int_0^a \frac{\sigma}{\varepsilon_0}\left(1 - z/a\right) dz$$

$$= \frac{l^2 \rho \sigma}{\varepsilon_0}\left[z - \frac{z^2}{2a} \right]_0^a$$

$$= \frac{l^2 \rho \sigma}{\varepsilon_0}\left[a - a/2 \right] = \frac{l^2 \rho \sigma a}{2\varepsilon_0} = \frac{-l^2 \sigma^2}{2\varepsilon_0}$$

This is exactly the force we get if the upper plane is modelled as a surface charge $-\sigma$ and the average field is used.



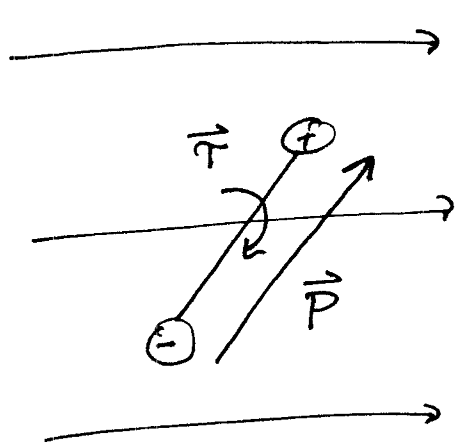
$$\vec{F} = -\sigma \vec{E}_{ave} = -\sigma \frac{\left(\frac{\sigma}{\epsilon_0} + 0\right)}{2}$$

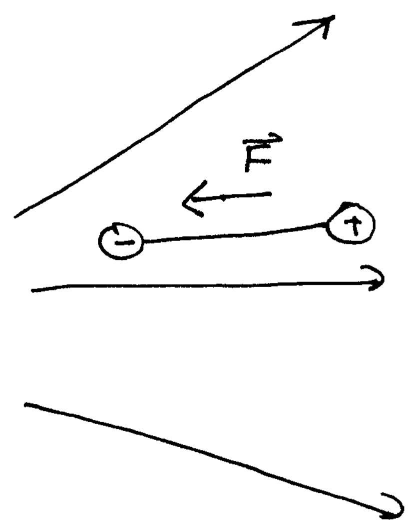
$$= \frac{-\sigma^2}{2\epsilon_0}$$

Note, when we use a surface charge or the line charge we are making the approximation that the details of the volume charge do not affect the physics.

Electric Dipole Mechanics (Chap 7.1)

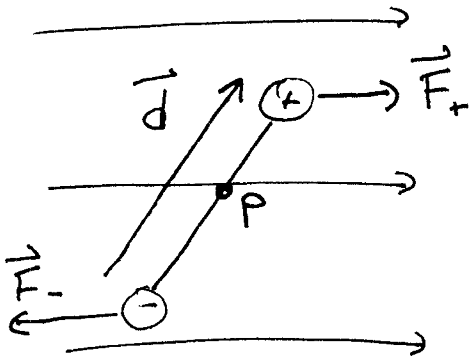
An electric dipole is an extended object, so it feels both a force on its center of mass and a torque about the center of mass. If the field is uniform, the net force is zero.





The torque (τ) about some point, P, is defined as $\vec{\tau} = \sum \vec{r} \times \vec{F}$ where $\vec{r}$ is a vector from the point P to the point where the force is applied.

If $\vec{E}$ changes slowly over the dipole



$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \frac{\vec{d}}{2} \times \vec{F}_+ + \left(-\frac{\vec{d}}{2}\right) \times \vec{F}_-$$

$$= \frac{\vec{d}}{2} \times (Q\vec{E}) + \left(-\frac{\vec{d}}{2}\right) \times (-Q\vec{E})$$

$$= Q\vec{d} \times \vec{E} = \vec{p} \times \vec{E}$$

$\boxed{\text{Torque on point dipole} - \vec{\tau} = \vec{p} \times \vec{E}}$

Now, let's find the force on the dipole.

The dipole moment is $\vec{p} = Q(d_x, d_y, d_z)$

The force in the x-direction,

$$F_x = Q\,\Delta E_x$$

$$F_x = \frac{\partial E_x}{\partial x} Q\Delta x + \frac{\partial E_x}{\partial y} Q\Delta y + \frac{\partial E_x}{\partial z} Q\Delta z$$

$$= p_x \frac{\partial E_x}{\partial x} + p_y \frac{\partial E_x}{\partial y} + p_z \frac{\partial E_x}{\partial z}$$

so $\vec{F} = (\vec{p} \cdot \nabla)\vec{E}$

Note, if $\vec{E}$ = constant, $\vec{F} = 0$. Only non-uniform fields exert non-zero forces on dipoles.