Linear Magnetic Materials

For some materials (not iron), if a material is placed in the field, a magnetic moment will be induced on the material in the same (or opposite) direction of the field.

If the relation between the applied field and the induced dipole is linear, we can write

$$\vec{M} = \chi_m \vec{H}$$

<u>Magnetic Susceptibility</u> (χ_m) - Constant characterizing magnetic response of material.

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0 \vec{H} + \mu_0 \chi_m \vec{H}$$

$$= \mu_0 (1 + \chi_m) \vec{H}$$

$$\mu_r \equiv 1 + \chi_m \implies \text{Relative Permeability}$$

$$\mu \equiv \mu_r \mu_0 \implies \text{Permeability}$$

Ampere's Law for Linear materials

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \left(\frac{\vec{B}}{\mu} \right) = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{B} = \mu \vec{J}_f + \mu \frac{\partial \vec{D}}{\partial t}$$

$$= \mu \vec{J}_f + \mu \varepsilon \frac{\partial \vec{E}}{\partial t}$$

If static,

$$\nabla \times \vec{B} = \mu \vec{J}_f$$

Most materials have a very small susceptability,

$$\chi_m(\text{copper}) = -1 \times 10^{-5}$$

$$\chi_m(\text{polyethylene}) = -0.2 \times 10^{-5}$$

$$\chi_m(\text{aluminum}) = 2.1 \times 10^{-5}$$

Some don't

$$\chi_m(\text{iron}) \sim 1000$$

$$\chi_m(\text{superconductor}) \equiv -1$$

(1) Note, both iron and the superconductor are very non-linear.

(2) Note, χ_m changes sign.

Paramagnetic $\chi_m > 1 \Rightarrow$ Moment same direction as field.

Diamagnetic $\chi_m < 1 \Rightarrow$ Moment opposite direction as field.

If $\chi_m > 1$, $\vec{M} = \chi_m \vec{H}$

$\Rightarrow \vec{M} \parallel \vec{H}$, the moment points in the same direction as the field.

If $\chi_m < 1$, $\vec{M} = \chi_m \vec{H}$, moment opposite field

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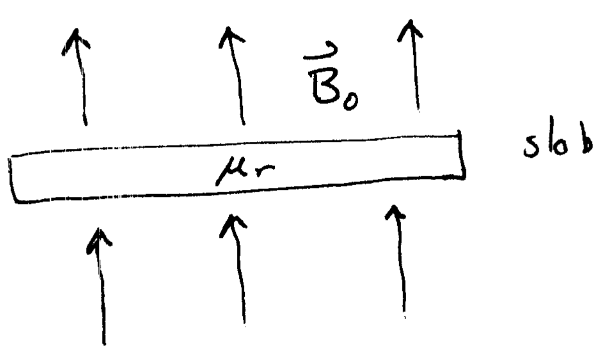
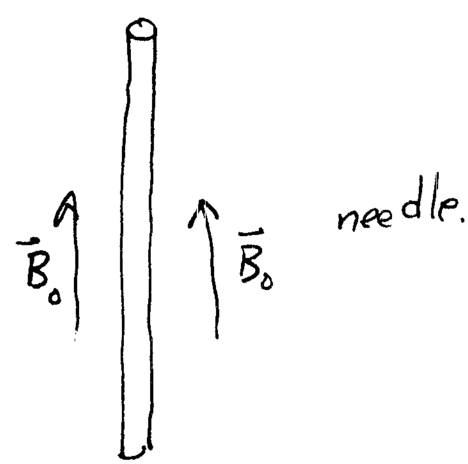
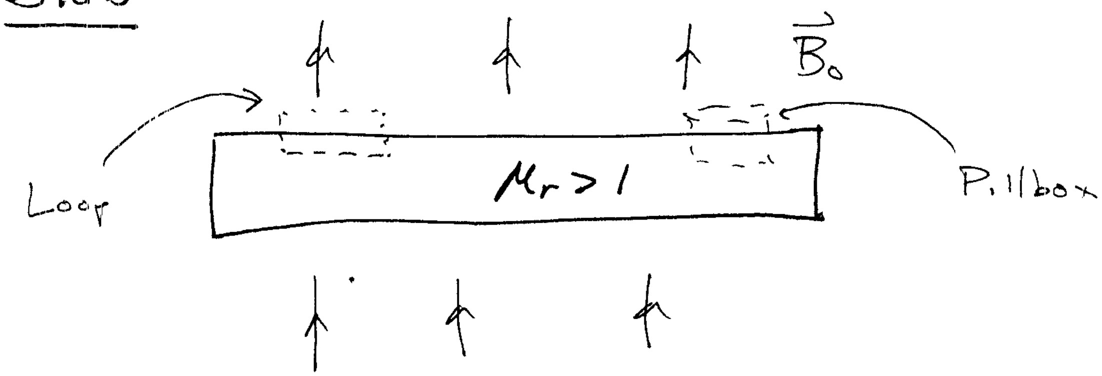
Let's put some magnetic materials in fields





Slab



Static

$$\nabla \times \vec{H} = \vec{J}_f \qquad\qquad \nabla \cdot \vec{B} = 0$$

– or –

$$\nabla \times \vec{B} = \mu_0 \vec{J}_t$$

Since $\vec{J}_f = 0$, $\nabla \times \vec{H} = 0$ and $\vec{H}$ = constant

This does not mean that $\vec{H}$ has the same value everywhere, since we have to match $\vec{H}$ at each boundary.

Stokesian Loop

$\nabla \times \vec{H} = 0 \implies H_t$ continuous, but $\vec{H} \perp d\vec{l}$ so who cares.

$$\oint \vec{H} \cdot d\vec{l} = 0$$

Gaussian Pillbox

$$B_o A - B_i A = Q_{menc} = 0$$

$$B_i = B_o$$ (No magnetic charge to shield the field)

$$\vec{B}_o = \mu_o \vec{H}_o \qquad (\vec{M}_o = 0)$$

$$\boxed{\vec{H}_o = \frac{\vec{B}_o}{\mu_o}}$$

$$\vec{B}_i = \mu_o \vec{H}_i + \mu_o \vec{M}_i = \mu_o \mu_r \vec{H}_i = \vec{B}_o$$

$$\boxed{\vec{H}_i = \frac{\vec{B}_o}{\mu_o \mu_r}}$$

Magnetization

$$\vec{M}_i = \chi_m \vec{H}_i = \frac{\chi_m}{\mu_r} \frac{\vec{B}_o}{\mu_o}$$

$$= \frac{\chi_m}{1+\chi_m} \frac{\vec{B}_o}{\mu_o}$$

Suppose $\mu_r = 1000$ (iron)

$\chi_m = 999$

and $\vec{B}_0 = 1\,mT\,\hat{z}$

$$|\vec{M}_i| = \left(\frac{999}{1000}\right)\frac{1\times10^{-3}\,T}{4\pi\times10^{-7}\,Tm/A}$$

$$= 800\,\frac{A}{m}$$

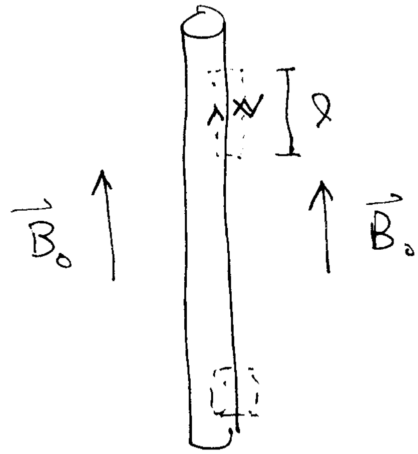
Total Dipole Moment $1\,m^2 \times 1\,cm$ thick sheet of iron.

$$m = |M|\,V = (800\,A/m)(1\,m^2)(0.01\,m)$$

$$= 8\,Am^2$$

$\Rightarrow$ Same moment you would get if 8 A flowed around $1m \times 1m$ loop.

Try Needle



Gaussian Pillbox

$B_{ot} = B_{it}$ (so what $\vec{B}_0 \perp \hat{n}$)

Stokesian Loop

$B_i l - B_o l = \mu_0 K l$

$\nabla \times \vec{B} = \mu_0 \vec{J}_t \implies \oint \vec{B} \cdot d\vec{l} = \mu_0 I_t$

or

$\oint \vec{H} \cdot d\vec{l} = 0 \implies \vec{H}_i = \vec{H}_o$

$$\vec{B}_o = \mu_o \vec{H}_o + \mu_o \vec{M}_o = \mu_o \vec{H}_o$$

$$\vec{H}_o = \frac{\vec{B}_o}{\mu_o} = \vec{H}_i$$

$$\vec{M}_i = \chi_m \vec{H}_i = \chi_m \frac{\vec{B}_o}{\mu_o}$$

$$\vec{B}_i = \mu_o \mu_r \vec{H}_i = \mu_o \mu_r \left(\frac{\vec{B}_o}{\mu_o} \right)$$

$$= \mu_r \vec{B}_o$$

So suppose nail ($\mu_r = 1000$) placed in Earth's magnetic field $B_o = 4 \times 10^{-5}\,T$. Inside nail we get $B_i = \mu_r B_o = 4 \times 10^{-2}\,T$, not bad.

Magnetization Density

$$M_i = \chi_m \frac{\vec{B}_o}{\mu_o} = (1000) \frac{4 \times 10^{-5}\,T}{4\pi \times 10^{-7}\,Tm/A}$$

$$= 32000 \frac{A}{m}$$

If the nail is 10cm long by 1mm in radius the dipole moment is

$$m = |\vec{M}_i| V = \left(32000 \frac{A}{m}\right)(0.1m)(0.001m)^2$$

$$= 0.003 \ Am^2$$

Surface Current

$$K = M \times \hat{n} = 32{,}000 \ A/m$$

or

$$B_i l - B_o l = \mu_o K l$$

$$B_i - B_o \approx B_i = \mu_o K$$

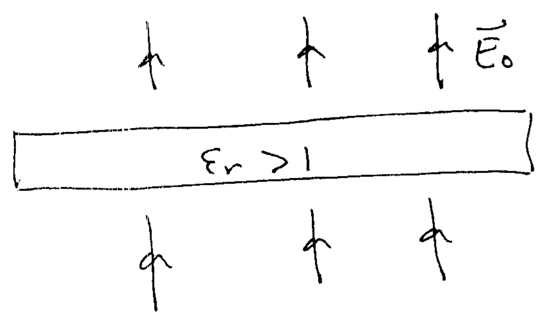
$$K = \frac{B_i}{\mu_o} = 1000 \frac{B_o}{\mu_o} = 32{,}000 \ A/m$$

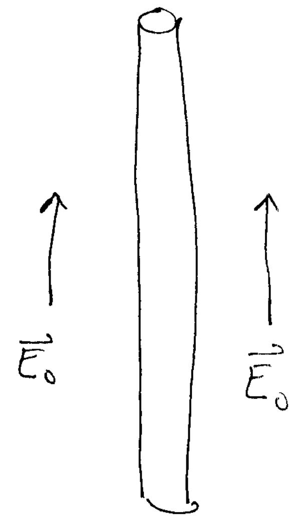
Why the big difference in responses?

The response is due to an effective surface current.

The slab minimizes the surface area for that current.

Recall Dielectrics





$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_r \varepsilon_0 \vec{E}$$

$$\nabla \cdot \vec{D} = \rho_f$$

$$\nabla \times \vec{E} = 0$$

Since $\rho_f = 0$,

$$\nabla \cdot \vec{D} = 0$$

$$\Rightarrow \vec{D}_o = \vec{D}_i$$

$$\vec{D}_o = \varepsilon_0 \vec{E}_0 \qquad \vec{D}_i = \varepsilon_0 \vec{E}_0$$

$$= \varepsilon_r \varepsilon_0 \vec{E}_i$$

$$\Rightarrow \quad \vec{E}_i = \frac{\vec{E}_0}{\varepsilon_r}$$

$$\sigma_b = \varepsilon_0 \left(E_0 - E_i\right)$$

$$= \vec{P} \cdot \hat{n}$$

$$\vec{P}_i = \chi_e \varepsilon_0 \vec{E}_i$$

$$= \frac{\chi_e \varepsilon_0 \vec{E}_0}{\varepsilon_r} = \frac{\chi_e}{(\chi_e + 1)} \varepsilon_0 \vec{E}_0$$

$$\sigma_b = \hat{n} \cdot \vec{P} = \frac{\chi_e}{(\chi_e + 1)} \varepsilon_0 E_0$$

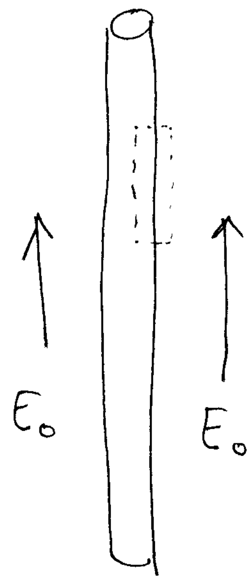
$$\varepsilon_0 \left(E_0 - E_i\right) = \varepsilon_0 \left(E_0 - \frac{E_0}{\varepsilon_r}\right)$$

$$= \varepsilon_0 \left(E_0 - \frac{E_0}{\chi_e + 1}\right)$$

$$= \frac{\varepsilon_0 E_0 \chi_e}{\chi_e + 1}$$

Stokesian Loop no useful info.

Needle



Gaussion surface no useful info

Stokesian Loop

$$\nabla \times \vec{E} = 0 \implies \oint \vec{E} \cdot d\vec{l} \implies E_i = E_0$$

$$D_0 = \varepsilon_0 E_0 \qquad D_i = \varepsilon_0 \varepsilon_r E_i = \varepsilon_0 \varepsilon_r E_0$$

$$P_0 = 0 \qquad P_i = \chi_e \varepsilon_0 E_0$$