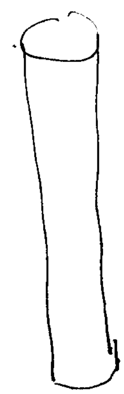
Magnetic Circuits

Let's build some magnets

We have already considered an infinitely long bar magnet.



$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = 0$$

$$\vec{H}_i = \vec{H}_o = 0$$

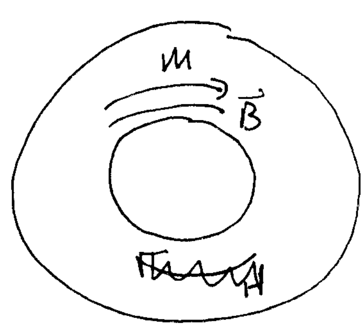
$$B_i = \mu_0 H_i + \mu_0 M$$

$$B_i = \mu_0 M$$

For NdFeB, $M = 1.02 \times 10^6 \, A/m$,

$$B_i = 1.28\,T$$

Let's bend the magnet into a circle so its managable



Now what's B_i?

$$\nabla \times H = 0$$

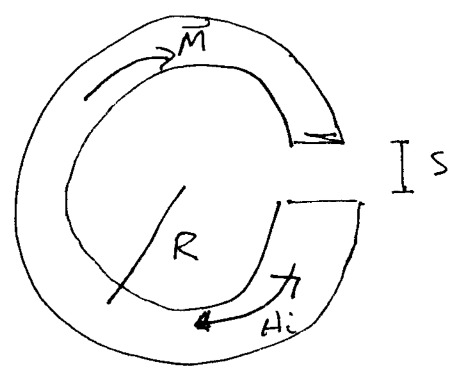
$$\oint \vec{H} \cdot d\vec{l} = 0 \implies H_i = 0 \implies B_i = \mu_0 M = 1.28\,T$$

magnetomotive force

$$\text{mmf} = \oint \vec{H} \cdot d\vec{l}$$

(not a force)

Problem   Can't use the magnet, because we can't get inside it. Solution - Cut a hole



Still no free currents so

$$\text{mmf} = \oint \vec{H} \cdot d\vec{l} = 0$$

Still no monopoles so $\nabla \cdot \vec{B} = 0 \implies B_o = B_i \equiv B$

$\cancel{H_0 = \mu_0 B}$ $\quad H_0 = B_0/\mu_0 = B/\mu_0$

$$B_i = B = \mu_0 H_i + \mu_0 M$$

$$\oint \vec{H}\cdot d\vec{l} = (2\pi R - s) H_i + s H_0 = 0$$

what direction does H_i point?

$\nabla \cdot \vec{H} = -\nabla \cdot M$ at interface

$H_0 - H_i = -(-M)$

$H_0 < H_i$

$\cancel{H_i}$ $\quad H_0 - H_i = M$

$\Longrightarrow H_i < 0$

$$H_i = \frac{B - \mu_0 M}{\mu_0}$$

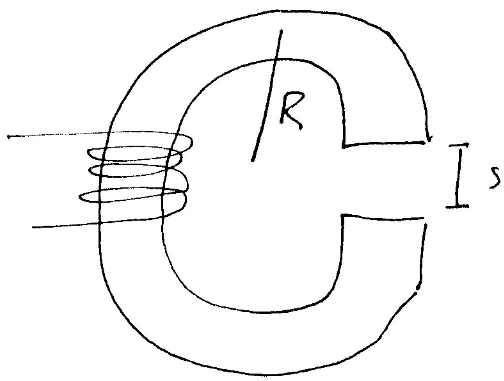
$$(2\pi R - s)\left(\frac{B - \mu_0 M}{\mu_0}\right) + s\,\frac{B}{\mu_0} = 0$$

$$2\pi R\left(\frac{B}{\mu_0}\right) = (2\pi R - s) M$$

$$B = \left(1 - \frac{s}{2\pi R}\right)\mu_0 M$$

$\Rightarrow$ The magnetic field is reduced by a factor of
the ratio of the gap size to the total path length.

<u>Ex</u> Electromagnet, N turns, radius R, gap s
iron μ_r



$B_i = B_o$ as before

$$\text{mmf} = \oint \vec{H}\cdot d\vec{l} = \cancel{I_f} = H_i(2\pi R - s) + sH_o = NI$$

$$H_o = \frac{B}{\mu_0} \qquad H_i = \frac{B}{\mu_0 \mu_r}$$

$$\frac{B}{\mu_0\mu_r}(2\pi R - s) + s\frac{B}{\mu_0} = NI$$

$$B\left(\frac{2\pi R}{\mu_r} - \frac{s}{\mu_r} + s\right) = NI$$

$$B\left(2\pi R - s + \mu_r s\right) = \mu_r NI$$ (~~Field no gap~~)

$$B\left(2\pi R + s(\mu_r - 1)\right) = \mu_r NI$$

$$B\left(1 + \frac{s(\mu_r - 1)}{2\pi R}\right) = \frac{\mu_r NI}{2\pi R}$$

$$B = \frac{1}{1 + \frac{s(\mu_r - 1)}{2\pi R}} \cdot \frac{\mu_r NI}{2\pi R}$$

So the field is down by a factor of

$$\frac{1}{1 + \frac{s(\mu_r - 1)}{2\pi R}}$$ from the non-gap field of $B = \frac{\mu_r NI}{2\pi R}$

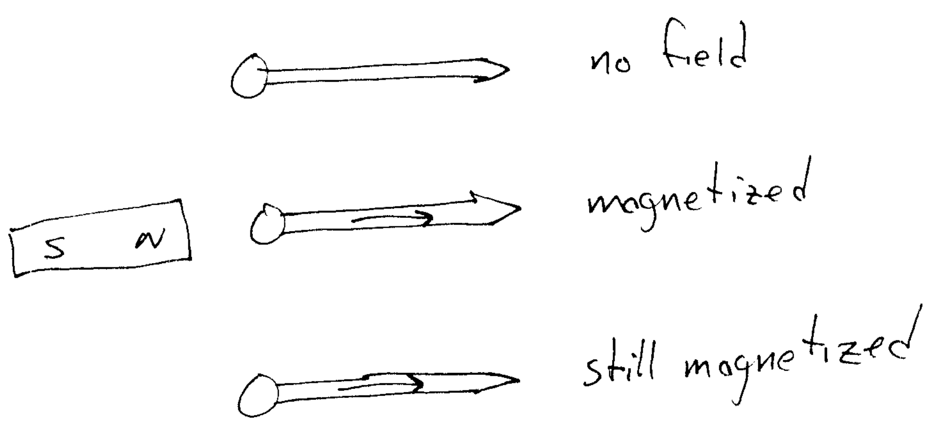
If the gap is $\frac{1}{10}\, 2\pi R$, the reduction is

$\cancel{\mu_r}\ \frac{10}{\mu_r - 1}$ which is large.

Ferromagnetism

Some materials have a gigantic magnetic response due to an energetic favorability to align spins over large areas of the material. The most common compound is iron, so the materials are called Ferromagnetic Materials.

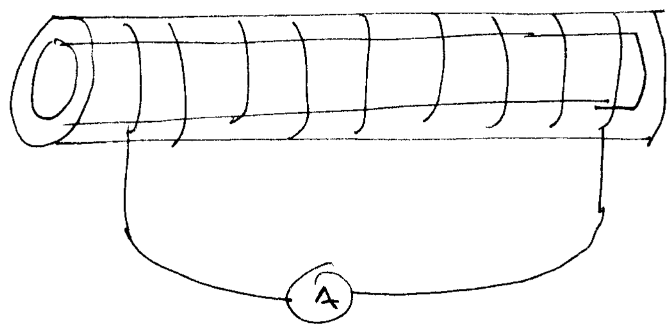
I had everyone magnetize a nail



So we have two states of magnetization corresponding to one state of applied field. ⇒ The ferromagnet remembers its history.

E-mail Rachae!

Let's build a device to investigate this.



Put an iron core in solenoid.

Solve for the fields, $\vec{J}_f = 0$

$$\Rightarrow \quad \nabla \times \vec{H}_f = 0$$

$\Rightarrow$ Same result as if the material was not present.

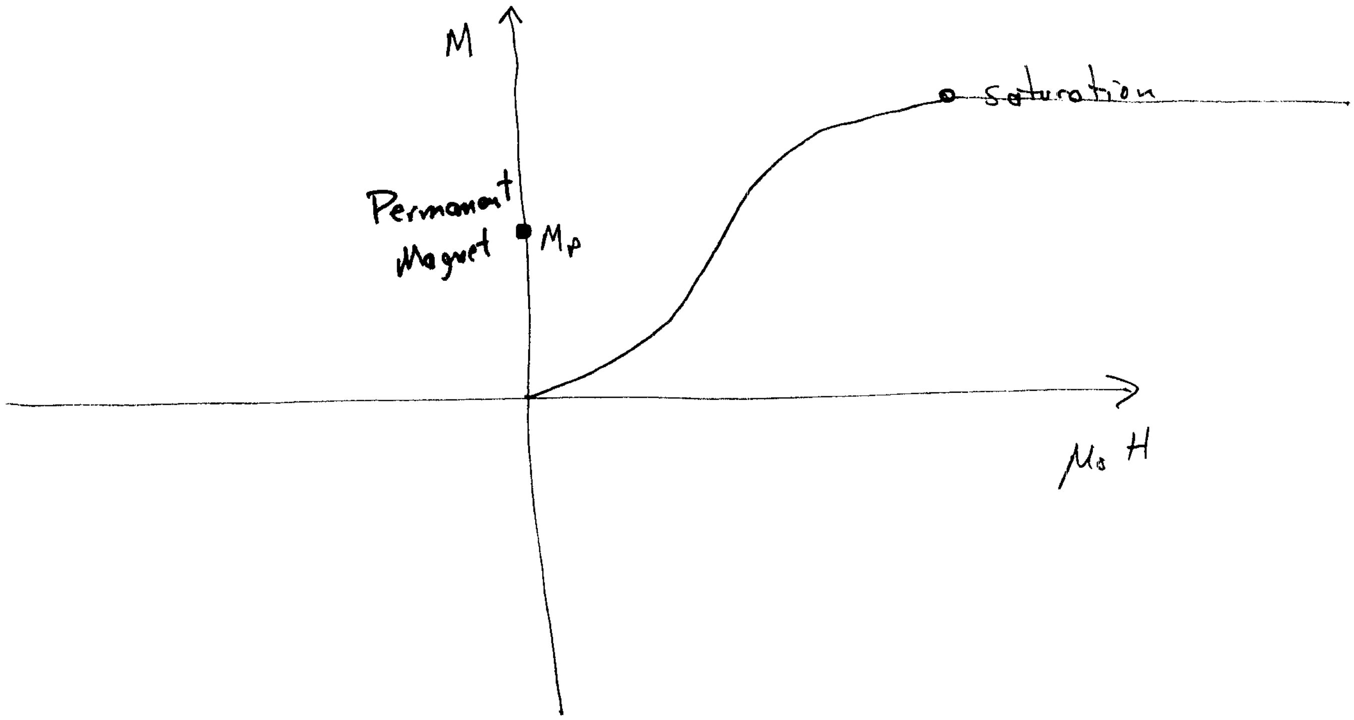
$$\oint \vec{H} \cdot d\vec{l} = I_f = N' I l = H_i l$$

$$H_i = N I' \qquad H_o = 0$$

$$B_i = \mu_o \mu_r H_i = \mu_r B_{air}$$

where B_{air} is field that would be produced in a air-filled solenoid. $B_{air} = \mu_o H_i$

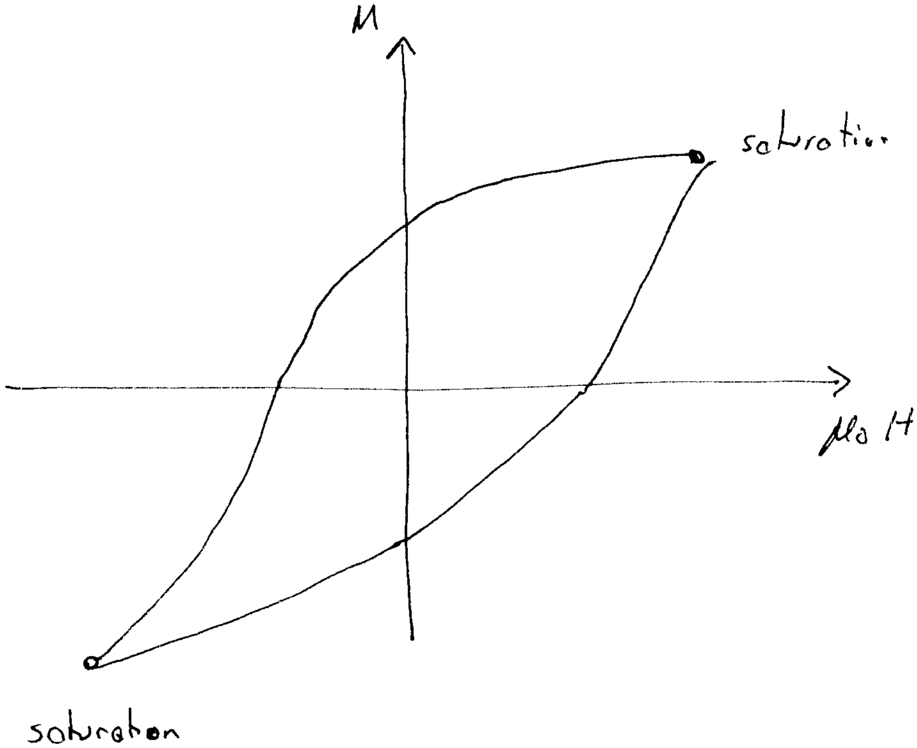
Lets Plot Magnetization against the applied field ($B_{air} = \mu_0 H$)



We reach a point where most applied field does not generate more magnetization. At this point, all the dipoles are aligned.

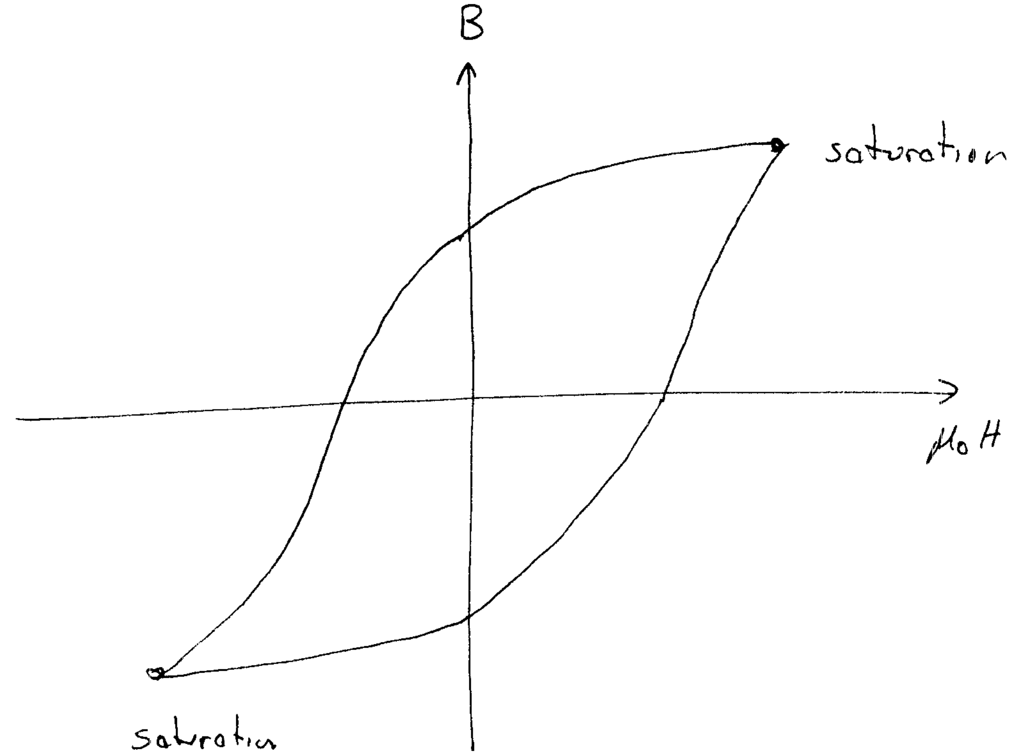
If we then turn off the applied field, the magnetization decreases but not to zero. At this point the material is a permanent magnet with magnetization density $\dot{M}_p$.

To erase this magnetization, we apply a reverse current. Eventually we can drive the magnetization through a cycle from saturation in one direction to saturation in the other direction



Now, plot this in terms of the magnetic field

$$\vec{B} = \underbrace{\mu_0 \vec{H}}_{\text{tiny}} + \mu_0 \vec{M}$$



This curve is called the <u>hysteresis loop</u>. It is different for every magnetic material.

Note, a magnetic device operating on AC power will be driven through this loop at the frequency of the device.

The energy lost in each cycle $\left(\frac{1}{2}\vec{B}\cdot\vec{H}\right)$ will be the area of the curve.

$\Rightarrow$ Heat destroys magnets - Each magnet has a temperature, the Curie temperature, where magnetization is lost