Magnetization - Magnetic Materials

Magnetization ($\vec{M}$) - or magnetization density

$\Rightarrow$ Magnetic dipole moment per unit volume

When we considered electric dipole moment per unit volume, we displaced two charged objects to form the dipole. That's not available in magnetism. We'll just have to do the math.

Dipole vector potential

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

Integrate over the density

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}\,') \times \hat{r}''}{r''^2} \, dv'$$

As before,

$$\nabla'\left(\frac{1}{r''}\right) = \frac{\hat{r}''}{r''^2}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \vec{M} \times \nabla'\left(\frac{1}{r''}\right) dv' \qquad \text{(vector identity)}$$

$$= \frac{-\mu_0}{4\pi}\int \nabla' \times \left(\frac{\vec{M}}{r''}\right) dv' + \frac{\mu_0}{4\pi}\int \frac{\nabla' \times \vec{M}}{r''} dv'$$

Curl Version of Divergence Thm

$$\vec{A} = \frac{\mu_0}{4\pi}\int_S \frac{\vec{M} \times \hat{n}'}{r''} da' + \frac{\mu_0}{4\pi}\int \frac{\nabla' \times \vec{M}}{r''} dv'$$

Define a volume current $\vec{J}_b = \nabla \times \vec{M}$

and a surface current $\vec{K}_b = \vec{M} \times \hat{n}$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_S \frac{\vec{K}_b}{r''} da' + \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}_b}{r''} dv'$$

Catalogue Currents

Total (Physical) Current $\vec{J}_t$

$$\vec{J}_t = \vec{J}_f + \vec{J}_b + \vec{J}_{pol}$$

$\vec{J}_b \equiv$ Bound Current

$$\vec{J}_b = \nabla \times \vec{M}$$

$\vec{J}_{pol} \equiv$ Polarization Current

$$\vec{J}_{pol} = \frac{\partial \vec{P}}{\partial t}$$

$\vec{J}_f \equiv$ Free Current – Current not produced by magnetization or polarization. The current you put in or $\vec{J}_f = \vec{J}_t - \vec{J}_b - \vec{J}_{pol}$

Displacement Current $\vec{J}_d = \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

$\Rightarrow$ Not a real current.

Displacement Current Density II

$$\vec{J}_{d,II} = \frac{\partial D}{\partial t} = \frac{\partial \vec{P}}{\partial t} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Ampere's Law

$$\nabla \times \vec{B} = \mu_0 \vec{J}_t + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$= \mu_0 \vec{J}_f + \mu_0 \vec{J}_b + \mu_0 J_{pol} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$= \mu_0 \vec{J}_f + \mu_0 \nabla \times \vec{M} + \mu_0 \frac{\partial \vec{P}}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{B} - \mu_0 \nabla \times \vec{M} = \mu_0 \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

Define $\vec{H}$ (called "H")

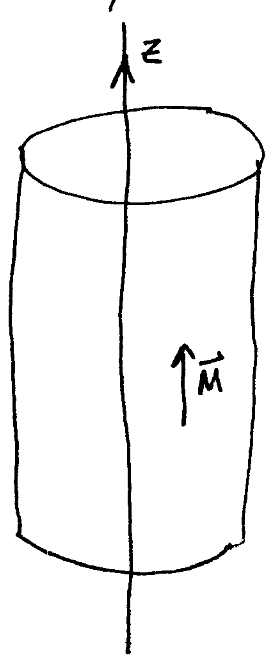
$$\mu_0 \vec{H} = \vec{B} - \mu_0 \vec{M} \implies \vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

Note about magnetic terminology - There is some argument (not among physicists) about whether $\vec{H}$ or $\vec{B}$ is the magnetic field. Some texts will call $\vec{H}$ the magnetic field and $\vec{B}$ something weird like magnetic flux density. The symbols however will be used consistently. $\vec{B}$ is the quantity that gives the magnetic force through the Lorentz force, $F = q\vec{v}\times\vec{B}$. It is the magnetic field.)

Let's consider some cases.

I. Infinite cylinder with constant magnetization $\vec{M}$. No free currents.



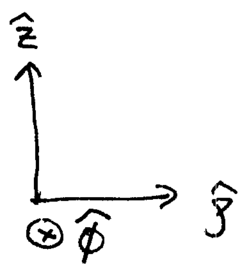
$$\vec{J}_b = \nabla \times \vec{M} = 0$$

$$\vec{K}_b = \vec{M} \times \hat{n}$$

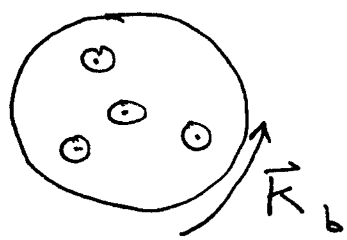
$$\vec{M} = M_0 \hat{z} \qquad \hat{n} = \hat{\rho}$$

$$\vec{K}_b = (M_0 \hat{z}) \times \hat{\rho}$$

$$= M_0 \hat{\phi}$$



End View



How much current?

For NdFeB, a common permanent magnet material $M_0 = 1.02 \times 10^6$ A/m $\Longrightarrow$ 10^6 A if magnet 1 m long.

At this point we can compute the field directly. We know all the currents, so we don't need $\vec{H}$.

For a cylindrical current $\vec{K}_b = M_0 \hat{\phi}$ we the same geometry as an infinite solenoid.

$$\left[\begin{array}{l} \vec{B}_i = \mu_0 K_b \hat{z} = \mu_0 M_0 \hat{z} \\ \\ \vec{B}_o = 0 \end{array}\right.$$

Now let's attack it through $\vec{H}$.

static so, $\nabla \times \vec{H} = \vec{J}_f = 0$

$$\Rightarrow \vec{H} = 0 \quad \text{since} \quad \vec{J}_f = 0.$$

<u>Outside</u>, $\vec{M} = 0$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = 0$$

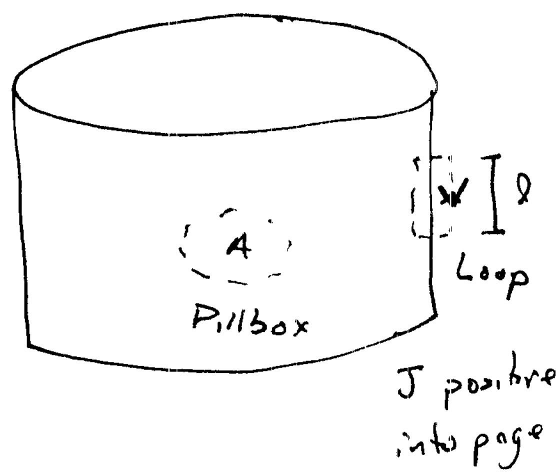
$$\Rightarrow \vec{B} = 0$$

Inside

$$\vec{B} = \mu_0 \vec{M} + \mu_0 \vec{H} = \mu_0 \vec{M}$$

Check Boundary Conditions

$$\nabla \cdot \vec{B} = \mu_0 (\nabla \cdot \vec{M} + \nabla \cdot \vec{H}) = 0$$



Pillbox

$$0 = \int \vec{H} \cdot d\vec{a} = \int \vec{M} \cdot d\vec{a} = 0$$

identically $\qquad \vec{M} \perp \hat{n}$

Loop

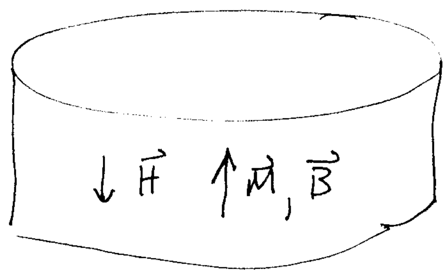
$$\nabla \times \vec{B} = \mu_0 J_t = \mu_0 \nabla \times \vec{H} + \mu_0 \nabla \times \vec{M}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J}_t \cdot d\vec{a} = \mu_0 \int \vec{H} \cdot d\vec{l} + \mu_0 \int \vec{M} \cdot d\vec{l}$$

$$B_i l = \mu_0 K l = 0 + \mu_0 M l$$

$$B_i = \mu_0 K = \mu_0 M \checkmark$$

Because we dealt with an infinite solenoid, $\vec{H}$ was zero. Suppose the solenoid was finite,



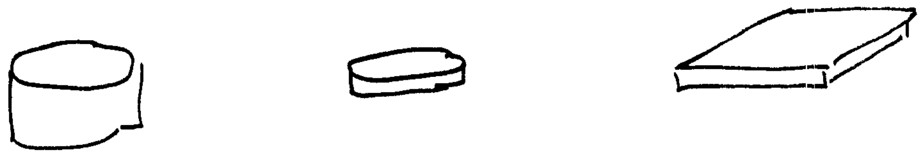
Use pillbox at surface,

$$\nabla \cdot \vec{B} = 0 = \mu_0 \nabla \cdot \vec{H} + \mu_0 \nabla \cdot \vec{M}$$

$$\nabla \cdot \vec{H} = -\nabla \cdot \vec{M} \implies \vec{H} \text{ points downward}$$

in permanent magnet.

Consider other magnet geometries



How can we calculate the field?

$\vec{K}_b = \vec{M} \times \hat{n}$ gives surface current