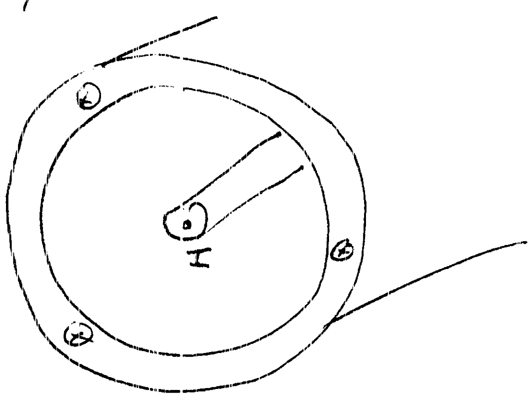
Ampere's Law (static)

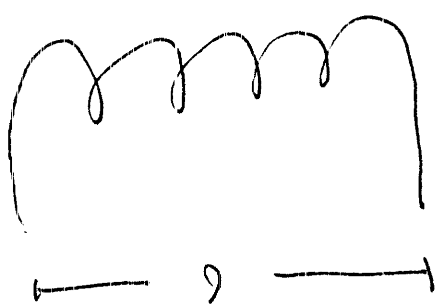
$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad \text{-or-} \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Useful calculation tool for:

(1) Cylindrical Systems



(2) Solenoids (infinite)

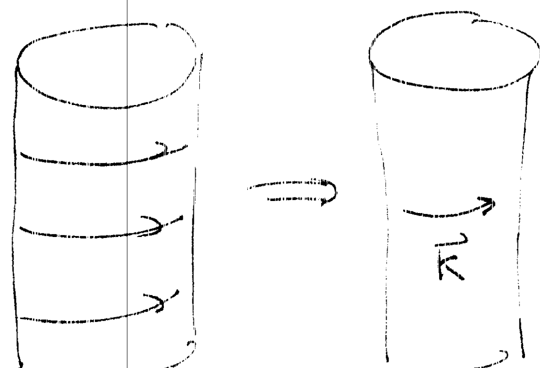


N turns (wraps)

$$B = \mu_0 \frac{N}{l} I$$

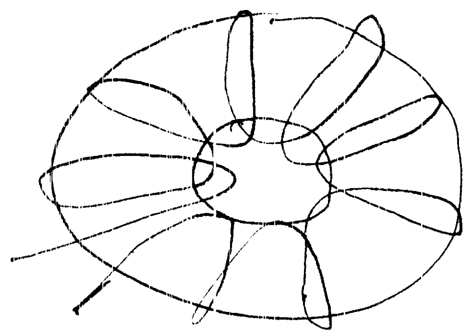
inside

$B = 0$ outside

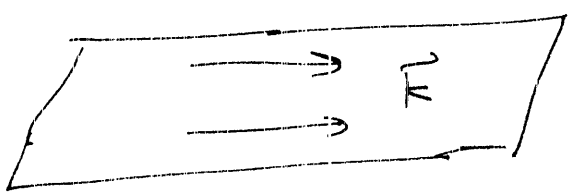


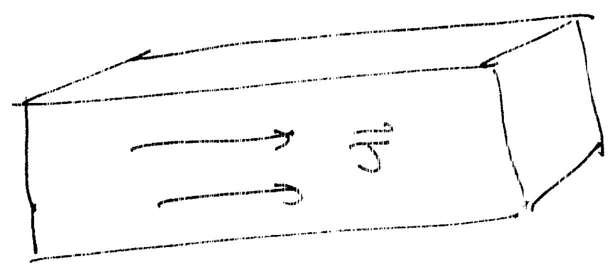
$$|\vec{K}| = \frac{N}{l} I$$

(3) Toroidal Solenoids

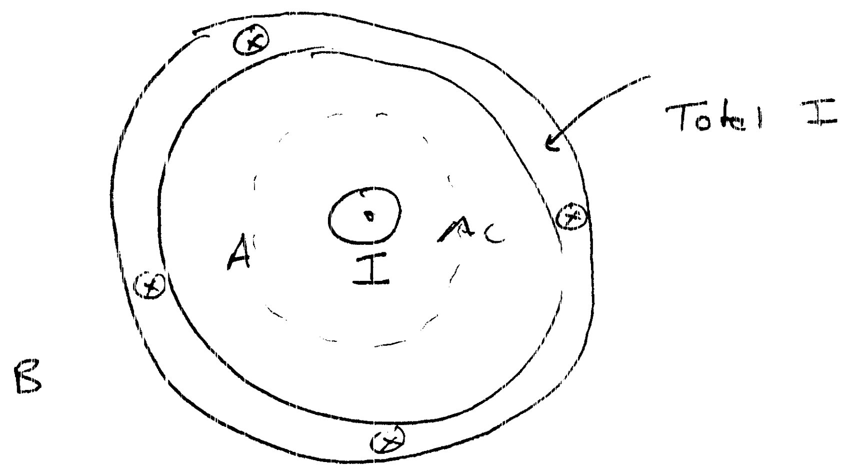


(4) Sheets or ~~R~~ Rectangular Volume Currents





Ex Co-axial Cable, inner radius a, outer radius b. Find field everywhere. Inner wire carries current I that returns in the outer wire.



By RHR for wire, magnetic field is counter-clockwise between the conductors.

Region A - Between the conductors, $I_{enc} = I$

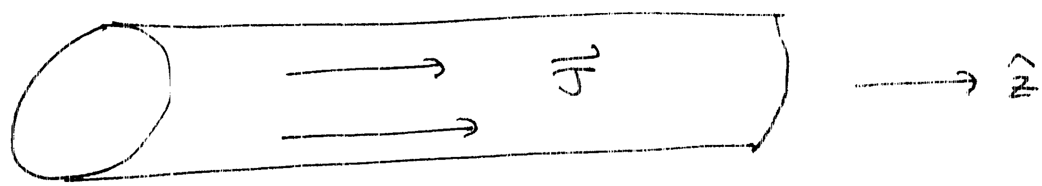
$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 I_{enc} = I$$

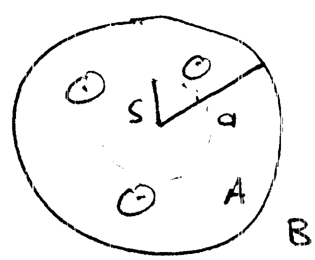
length of path

$$\vec{B}_A = \frac{\mu_0 I}{2\pi r} \quad \text{counter-clockwise}$$

Region B $\quad I - I = I_{enc} = 0$

Ex A cylindrical region contains a non-uniform current density $\vec{J} = J_0 s \hat{z}$ for $r < a$. Compute the magnetic field everywhere.





Region A $\quad s < a$

$$I_{enc} = \int_S \vec{J} \cdot \hat{z}\, da = \int_S J_0 s\, da$$

$$= \int_0^{2\pi}\int_0^{s} J_0 s^2\, ds\, d\phi \qquad da = ds\, s\, d\phi$$

$$I_{enc} = 2\pi J_0 \int_0^{s} s^2\, ds$$

$$= \frac{2}{3}\pi J_0 s^3$$

<u>Ampere's Law</u>

$$\oint \vec{B}\cdot d\vec{l} = 2\pi s B = \mu_0 I_{enc}$$

$$B_A = \frac{\mu_0 I_{enc}}{2\pi s} = \frac{\frac{2}{3}\pi J_0 s^3}{2\pi s}$$

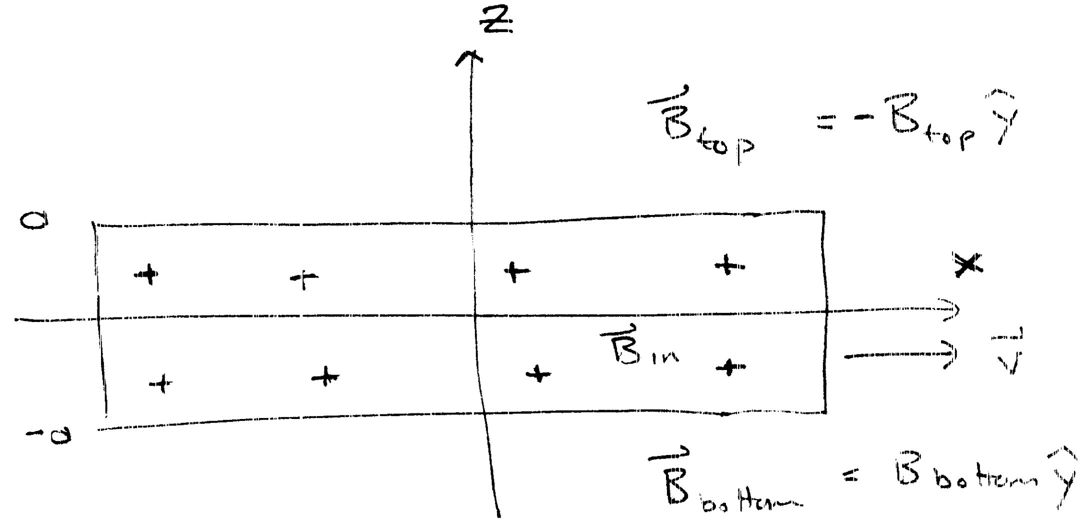
$$\vec{B}_A = \frac{1}{3} J_0 s^2 \quad \text{counter-clockwise} \quad \text{(RHR wire)}$$

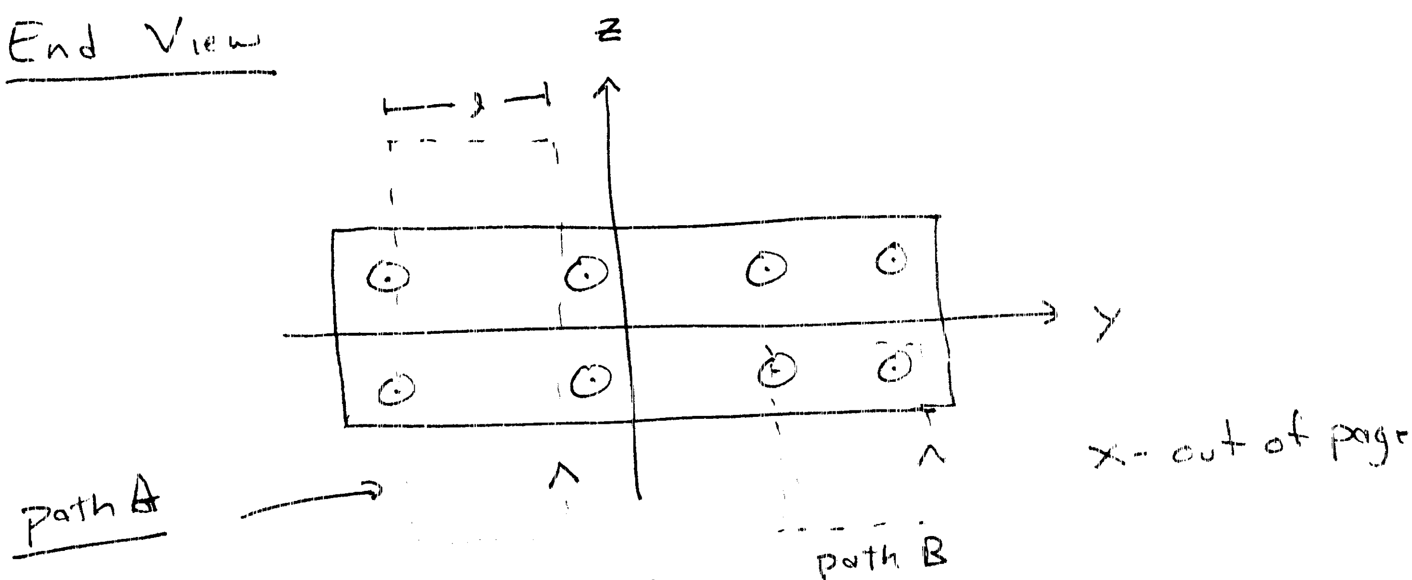
<u>Region B</u> $\quad s > a \qquad I_{enc} = \frac{2}{3}\pi J_0 a^3$

$$\vec{B}_B = \frac{\mu_0 I_{enc}}{2\pi s} = \frac{\frac{2}{3}\pi J_0 a^3}{2\pi s}$$

$$= \frac{J_0 a^3}{3 s} \quad \text{counter-clockwise}$$

Ex A slab with uniform volume charge density ρ occupies the region between $z = -a$, and $z = a$. The slab moves with velocity $\vec{v} = v_0 \hat{x}$. Compute magnetic field.





$$\vec{J} = \rho\vec{v} = \rho v_0 \hat{x}$$

Recall, $\vec{B}$ is a psvedo-vector which changes sign under reflection or test direction by applying RHR to $d\vec{B} = \mu_0 I \frac{d\vec{l} \times \hat{r}}{r^2}$.

Field points in $-y$ direction above slab and $+y$ direction below slab.

<u>Apply Ampere's Law to Path A</u>

$$d\vec{l}_{top} = -dy\,\hat{y} \qquad d\vec{l}_{bottom} = dy\,\hat{y}$$

$$\oint \vec{B}\cdot d\vec{l} = \vec{B}_{top}\cdot(-\hat{y})\,l + \vec{B}_{bottom}\cdot(\hat{y})\,l$$

$$= B_{top}\,l + B_{bottom}\,l$$

By symmetry $B_{top} = B_{bottom} = B_{out}$

$$2B_{out}\,l = \mu_0 I_{enc} = \mu_0 (J \cdot 2a\,l)$$

$$= \mu_0 \rho v_0 2a\,l$$

$$B_{out} = \mu_0 \rho v_0 a$$

$$\vec{B}_{top} = -\mu_0 \rho v_0 a \,\hat{y} \qquad \vec{B}_{bottom} = \mu_0 \rho v_0 a \,\hat{y}$$

<u>Apply Ampere's Law to Path B</u> Let z be the location of the top of the path and

$$B_{in}(z)\,\hat{y}$$

be the field.

$$\oint_{path\,2} \vec{B}\cdot d\vec{l} = B_{bottom}\, l - B_{top}\, l = \mu_0 I_{enc}$$

$$I_{enc} = J \cdot l \cdot (z+a) = J\, l\,(z+a)$$

$$= \rho v_0 l (z+a)$$

$$B_{top} = B_{bottom} - \mu_0 \rho v_0 (z+a)$$

$$= \mu_0 \rho v_0 a - \mu_0 \rho v_0 (z+a)$$

$$\vec{B}_{top} = -\mu_0 \rho v_0 z \,\hat{y}$$