Conduction

The electromagnetic field in a conductor exerts a force on the mobile charge to create a current.

Under the <u>assumption</u> of linearity

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$$

Under most circumstances, the $\vec{E}$ term is dominant.

<u>Ohm's Law</u>

$$\vec{J} = \sigma \vec{E}$$

<u>Conductivity</u> (σ) - Intrinsic property of conductors controling the amount of current.

- Units $[\sigma] = \frac{S}{m}$ $\quad$ $S \equiv$ Sieman
- $1\,S = \frac{1}{\Omega}$
- Ohm $1\,\Omega = 1\,V/A$
- <u>Archaic</u> $1\,S = 1\,\text{mho} = 1\,\mho$

Resistivity (ρ) - Intrinsic property of material that resists current flow

$$\rho = \frac{1}{\sigma}$$

- $[\rho] = \Omega \cdot m$

Resistance (R)

$$R \equiv \frac{\Delta V}{I}$$

$$[R] = \Omega$$

Power (P) - Power transferred from the field to the current.

$$P = \vec{F} \cdot \vec{v} = Q \vec{E} \cdot \vec{v}$$

velocity

Power per unit volume

$$\frac{dP}{dV} = \frac{Q}{V} \vec{E} \cdot \vec{v} = \rho \vec{E} \cdot \vec{v}$$

volume   charge density

$$\vec{J} = \rho \vec{v}$$

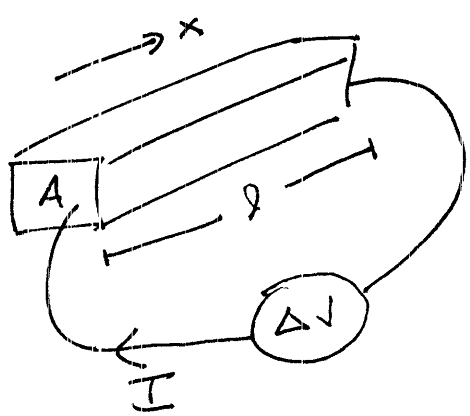
$$\frac{dP}{dV} = \vec{E} \cdot \vec{J} = \sigma \vec{E}^2 = \frac{J^2}{\sigma}$$

This power is converted into heat in the material.

Dfn <u>Joule Heating</u> - Power dissipate in a conductor

$$\frac{dP}{dV} = \vec{E} \cdot \vec{J}$$

<u>Ex</u> Consider a block of material with cross-sectional area A, length l. A potential ΔV across the ends causes a current I to flow.



The resistance is $R = \dfrac{\Delta V}{I}$

Assume a uniform current down the bar

$$\vec{J} = J_0 \hat{x}$$

By Ohm's Law, $\vec{E} = \dfrac{\vec{J}}{\sigma} = \dfrac{J_0}{\sigma} \hat{x}$

$\Rightarrow$ Note, if we know the current then we know the field.

The potential difference is as always,

$$|\Delta V| = \left| -\int \vec{E} \cdot d\vec{l} \right| = \frac{J_0}{\sigma} l$$

The total current is

$$I = \int J\, da = J_0 A$$

<u>Resistance</u>

$$R = \frac{\Delta V}{I} = \frac{J_0 l/\sigma}{J_0 A} = \frac{l}{\sigma A}$$

or in terms of resistivity

$$R = \frac{\rho l}{A}$$

Power converted to thermal energy

$$P = \left(\frac{dP}{dV}\right) V = \left(\frac{dP}{dV}\right) A l$$

volume

power per volume

$$P = \left(\frac{J_0^2}{\sigma}\right) A l = \left(\frac{J_0 l}{\sigma}\right) \cdot J_0 A$$

$$= \Delta V \, I$$

Power Dissipated in Electric Device

$$P = I \Delta V$$ Always

Sizes

	ρ
Copper	$1.7 \times 10^{-8}\ \Omega m$
Graphite	$1.4 \times 10^{-5}\ \Omega m$
Salt Water	$4.4 \times 10^{-2}\ \Omega m$
Pure Water	$2.5 \times 10^{5}\ \Omega m$
Glass	$10^{10} - 10^{14}\ \Omega m$

Ex Now suppose the conductivity changes along the bar, perhaps because of a temperature gradient.

$$\sigma(x) = \gamma (x + x_0)$$

Sln Assume steady current $\vec{J} = J_0 \hat{x}$ otherwise a net charge ~~be~~ continuously builds up in the material.

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{J_0}{\gamma(x+x_0)} \hat{x}$$

Potential Difference

$$|\Delta V| = \left| - \int \vec{E} \cdot d\vec{l} \right|$$

$$= \left| - \int_0^l \frac{J_0}{\gamma(x+x_0)} dx \right|$$

$$= \frac{J_0}{\gamma} \ln\left(\frac{l + x_0}{x_0} \right)$$

Total Current

$$I = J_0 A$$

Resistance

$$R = \frac{|\Delta V|}{I} = \frac{1}{\gamma A} \ln\left(\frac{l + x_0}{x_0} \right)$$

Note, since $\vec{J}$ changes with position, and therefore $\vec{E}$ changes with position, we have a static charge density

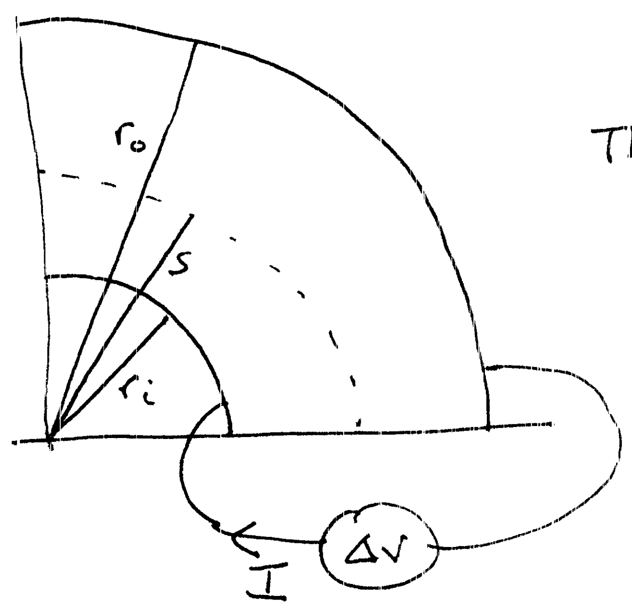
$$\nabla \cdot \vec{E} = \rho/\varepsilon_0$$

$$\rho = \varepsilon_0 \nabla \cdot \vec{E}$$

$$= \varepsilon_0 \frac{d}{dx} \frac{J_0}{\gamma(x+x_0)}$$

$$= -\frac{J_0 \varepsilon_0}{\gamma(x+x_0)^2}$$

Ex Consider the resistance of a cylindrical wedge of thickness a, inner radius r_i and outer radius r_o



Thickness a

The total current I through any cross-section must be constant, or charge progessively builds up.

The area of a cross-section of the wedge is

$$A(s) = \frac{\pi}{2} s a$$

where s is the radius and a the thickness.

Let I be the current provided by the supply.

$$I = J_i A(r_i) = J_o A(r_o)$$

The current density is then

$$\vec{J} = \frac{I}{A(s)} \hat{s} = \frac{2I}{\pi s a} \hat{s}$$

The field is, by Ohm's Law,

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{I}{\sigma A(s)} \hat{s}$$

$$= \frac{2I}{\sigma \pi a s} \hat{s}$$

The potential difference across the wedge is

$$\Delta V = -\int \vec{E} \cdot d\vec{l} = -\int_{r_i}^{r_o} \frac{2I}{\sigma \pi a s}\, ds$$

$$= -\frac{2I}{\sigma \pi a} \ln\left(\frac{r_o}{r_i}\right)$$

The resistance is

$$R = \frac{\Delta V}{I} = \frac{2}{\sigma \pi a} \ln\left(\frac{r_o}{r_i}\right)$$

<u>Power Dissipated in Wedge</u>

$$\frac{dP}{dV} = \vec{E} \cdot \vec{J} = \frac{J^2}{\sigma} = \frac{I^2}{\sigma A^2}$$

$$\text{Power} = P = \int \frac{dP}{dV}\, d\tau = \cancel{\frac{I^2}{\sigma A^2}}$$

$$= a \int_0^{\pi/2} d\phi \int_{r_i}^{r_o} ds\, s\, \frac{dP}{dV}$$

$$= \frac{\pi}{2} a \int_{r_i}^{r_o} ds\; s\, \frac{I^2}{\sigma\left(\frac{\pi}{2} s a\right)^2}$$

$$P = \frac{2}{\pi} \frac{1}{\sigma a} I^2 \int_{r_i}^{r_o} \frac{ds}{s}$$

$$P = \frac{2 I^2}{\pi a \sigma} \ln\left(\frac{r_o}{r_i}\right) = I^2 R \checkmark$$