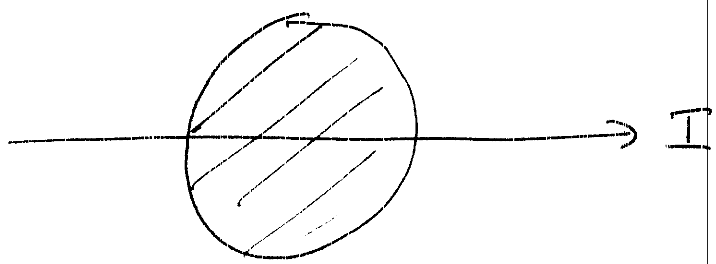
Current

Current (I) - Charge per unit time flowing through a surface S.



$\Rightarrow$ Units Amps $\quad 1A = 1C/s$

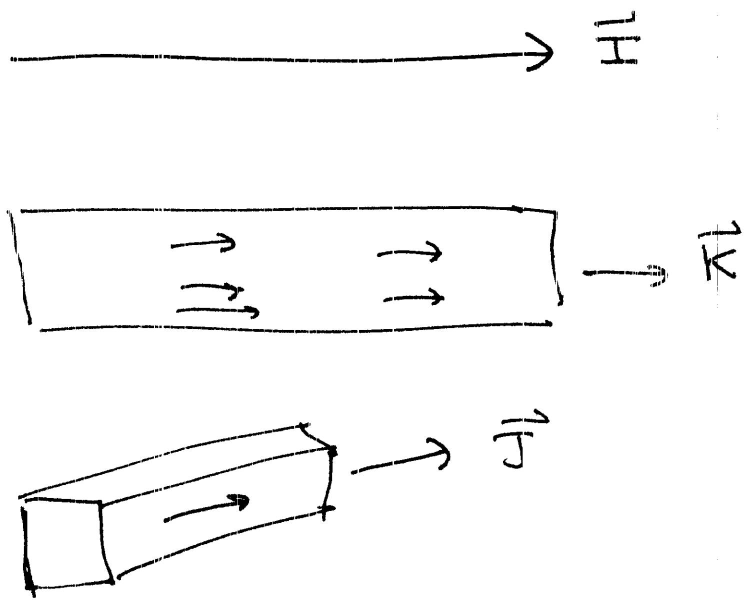
$\Rightarrow$ Define vector current $\vec{I} = I\hat{n}$ where $\hat{n}$ is the local direction of flow.

<u>Current Density</u> ($\vec{J}$) - The charge per unit area per unit time flowing in the direction $\hat{n}$.

$$\vec{J} = \frac{dI}{da}\hat{n} = \frac{d\vec{I}}{da}$$

<u>Surface Current Density</u> ($\vec{K}$) - Current per unit length flowing in direction $\hat{n}$.

$$\vec{K} = \frac{dI}{d\ell}\hat{n}$$



Total Current (I)

$$I = \int_S \vec{J}\cdot d\vec{a} = \int_S \vec{J}\cdot\hat{n}\,da$$

$$= \cancel{\int_c \vec{K}\cdot d\vec{l}}$$

$$I = \int_c \vec{K}\cdot\hat{n}\,dl$$

$\hat{n}$ is normal to c.

(Local) Conservation of Charge - The time rate of change of the net charge in a volume must equal the total flow of charge into the volume.

$$I_{in} = -\int_S \vec{J}\cdot\hat{n}\, da = \frac{d}{dt}\int_V \rho\, d\tau$$

outward normal

Divergence Thm

$$-\int_S (\nabla\cdot\vec{J})\, d\tau = \frac{d}{dt}\int \rho\, d\tau$$

for all V.

$$-\nabla\cdot\vec{J} = \frac{\partial \rho}{\partial t}$$

Continuity Eqn - Expresses conservation of charge.

$$\nabla\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Conservation of Charge is not external to Maxwell's Eqns.

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Take divergence of Ampere's Law

$$\nabla \cdot (\nabla \times \vec{B}) = 0 = \mu_0 \nabla \cdot \vec{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \nabla \cdot \vec{E}$$

identity

Use Gauss' Law

$$0 = \mu_0 \nabla \cdot \vec{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \rho/\varepsilon_0$$

$$\Longrightarrow \quad \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

We can create currents by allowing charge densities to move at velocity $\vec{v}$.

$\vec{J} = \rho \vec{v}$ — volume charge density moves with velocity $\vec{v}$.

$\vec{K} = \sigma \vec{v}$ — surface charge density moves with velocity $\vec{v}$.

$\vec{I} = \lambda \vec{v}$ — linear charge density moves with velocity $\vec{v}$.

<u>Ex</u> <u>Problem 5.6</u> — A thin disk has surface charge density σ and is rotated at angular velocity ω. Compute $\vec{K}$.

<u>Sln</u>

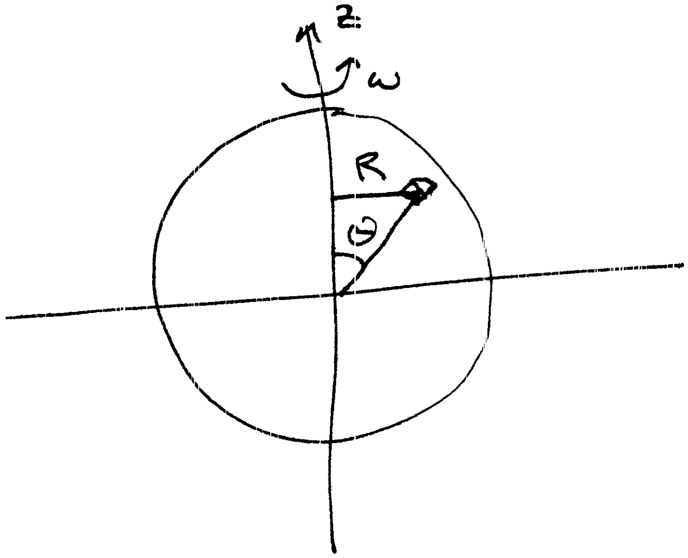
$$\vec{K} = \sigma \vec{v}$$

$$\vec{v} = s \frac{d\phi}{dt} \hat{\phi} = s\omega \hat{\phi}$$

$$\vec{K} = \sigma s \omega \hat{\phi}$$

Ex Spin a uniformly charged sphere with volume charge density ρ with angular velocity $\vec{\omega} = \omega \hat{z}$.

Sln



$$\vec{J} = \rho \vec{v}$$

$$\vec{v} = R\omega \hat{\phi} = r \sin\theta \, \omega \hat{\phi}$$

$$\vec{J} = \rho r \sin\theta \, \omega \hat{\phi}$$

Note, we can also write a current as

$\vec{I} = \sum q_i \vec{v}_i$, but restrictions are required to produce a steady current.