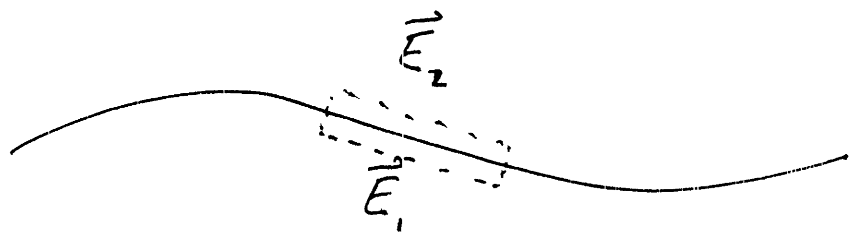
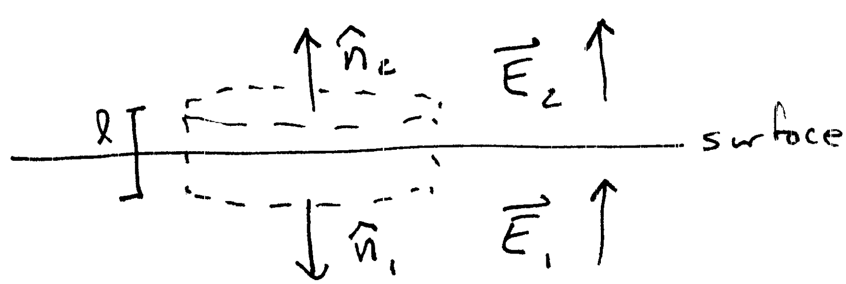
Electrostatic Boundary Conditions

Consider any surface and let $\vec{E}_1, V_1$ and $\vec{E}_2, V_2$ be the field and potential on the two sides of the surface.



Use a Gaussian surface, a Gaussian pillbox, with faces on opposite sides of the surface.



If ℓ becomes very small, there is no flux out the sides of the Gaussian surface.

Apply Gauss' Law

$$\Phi = \vec{E}_2 \cdot \hat{n}_2 A + \vec{E}_1 \cdot \hat{n}_1 A = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

~~from direction of [illegible] normal~~

where σ is any surface charge at the surface.

~~E~~ Let the upward normal be $\hat{n}$; $\hat{n}_2 = \hat{n}$, $\hat{n}_1 = -\hat{n}$

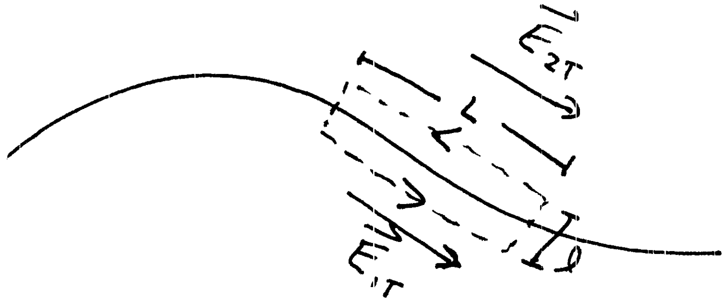
$$\Phi = \vec{E}_2 \cdot \hat{n} - \vec{E}_1 \cdot \hat{n} = \frac{\sigma}{\varepsilon_0}$$

$E_{in} = \vec{E}_i \cdot \hat{n}$ is the component of the field normal to the surface.

$$E_{2n} - E_{1n} = \frac{\sigma}{\varepsilon_0} \implies$$ The normal component of the field is discontinuous by σ/ε_0

Now apply Stoke's Law Chose a Stoksion

Loop, also s.t. l becomes small.



Let E_{1T} and E_{2T} be the tangential components of the field

$$\text{Since} \quad \nabla \times \vec{E} = 0 , \quad \oint_C \vec{E} \cdot d\vec{l} = E_{1T} L - E_{2T} L = 0$$

$$\Rightarrow \quad E_{1T} = E_{2T}$$

$\Rightarrow$ Tangential component of the field continuous.

$\Rightarrow$ The Gaussian pillbox and Stokesion (Amperion) path is one ~~other~~ of the best tricks in the book.

The two expression can be combined in a single equation

$$\vec{E}_2 - \vec{E}_1 = \frac{\sigma}{\varepsilon_0}\hat{n}$$

[$\hat{n}$ points from 1 to 2]

Since the potential is the integral of the field, the potential is continuous across the surface.

Since

$$\vec{E}_2 - \vec{E}_1 = \frac{\sigma}{\varepsilon_0}\hat{n}$$

$$-\nabla V_2 - (-\nabla V_1) = \frac{\sigma}{\varepsilon_0}\hat{n}$$

$$\hat{n}\cdot\nabla V_2 - \hat{n}\cdot\nabla V_1 = -\frac{\sigma}{\varepsilon_0}$$

$$\frac{\partial V_2}{\partial n} - \frac{\partial V_1}{\partial n} = -\frac{\sigma}{\varepsilon_0}$$

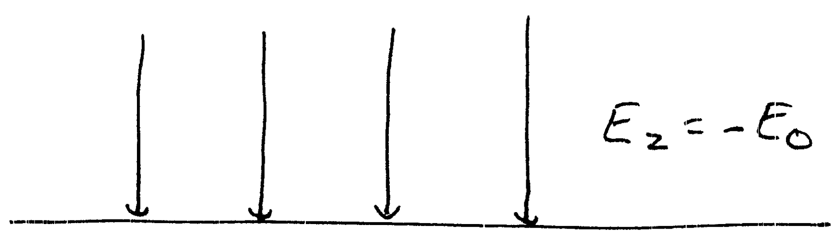
$\Rightarrow$ The derivative of the potential changes by $-\sigma/\varepsilon_0$

Where I have used the fact that the derivative of the potential in a some direction r is $\frac{\partial V}{\partial r} = \nabla V \cdot \hat{r}$

For example, $\frac{\partial V}{\partial x} = \nabla V \cdot \hat{x}$

so $\frac{\partial V}{\partial n} = \nabla V \cdot \hat{n}$

So what about our conductor



$E_z = -E_0$

$E_1 = 0$

$-E_0 - (0) = \frac{\sigma}{\varepsilon_0}$ $\quad\quad \sigma = -\varepsilon_0 E_0$

$P = \frac{1}{2}\varepsilon_0 E_0^2$ from previous pressure