Electricity and Magnetism - Final Exam - Spring 2010

Work four of the six problems. Place the problems in the order you wish them graded. The first two problems form the first half test; the second two problems form the second half test.

Problem 4.1 Show how the divergence of the Poynting vector,

$$\nabla \cdot \vec{S} = \nabla \cdot \frac{\vec{E} \times \vec{B}}{\mu_0}$$

is related to the electric and magnetic energy densities $u_e = \frac{1}{2}\epsilon_0 \vec{E} \cdot \vec{E}$ and $u_m = \frac{1}{2}\vec{B} \cdot \vec{B}/\mu_0$. Cite any law or vector identity used. Note, you do not need to know anything about the Poynting vector to do this problem. Assume $\vec{J} = 0$.

Problem 4.2 The $x - y$ plane is the boundary between two regions $z > 0$ and $z < 0$ with different electric and magnetic fields. The fields below the plane are

$$\vec{E}_- = \gamma(\hat{x} + 2\hat{y})$$

and

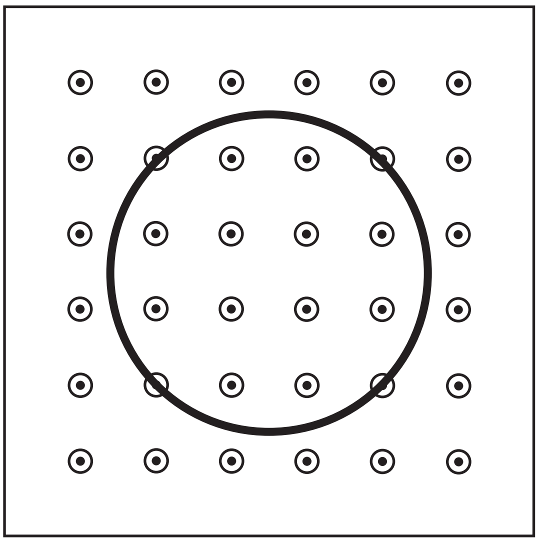
$$\vec{B}_- = \alpha(\hat{x} + \hat{z})$$

The plane has a surface charge density σ and a surface current density $\vec{K} = \Gamma\hat{y}$. Γ, γ, and α are constants. Find the electric and magnetic field above the place ($z > 0$).

Problem 4.3 A solenoid of radius b has N turns wound over ℓ distance and carries current I. The solenoid contains a hollow cylindrical iron core with relative permeability μ_r at the operating field. The iron core has inner radius a and outer radius b. Compute $\vec{B}$, $\vec{H}$, $\vec{M}$, $\vec{J}_b$, and $\vec{K}_b$ everywhere.

Problem 4.4 A spherical system of radius a has potential $V(a, \theta) = \gamma\cos(\theta)$ at its surface. Compute the electric field inside the system.

Problem 4.5 A circular ring of conducting wire is in a region with changing magnetic field as shown below. The radius of the ring is increasing as $a(t) = a_1 t^2$, where a_1 is constant. The magnetic field is $B(t) = B_0 t^2$ in the direction drawn with B_0 constant. The conductivity of the wire is σ and it has cross-sectional area A_w. Compute the current flowing the the wire as a function of time and give its direction.



Problem 4.6 A spherical system is built with a free charge density $\rho(r) = \gamma r$ $(r < a)$ embedded in a linear dielectric material of radius a with relative permittivity ϵ_r. Calculate the potential difference between the center of the system and the surface ΔV_{0a}. γ is constant.