The "H" field

The bound currents behave just like currents in Ampere's law –

$$\nabla \times \vec{B} = \mu_0 (\vec{J}_f + \vec{J}_b)$$

where $\vec{J}_f$ is the free current, any current that is not accounted for by bound current.

$$\nabla \times \frac{\vec{B}}{\mu_0} - \vec{J}_b = \vec{J}_f$$

$$\nabla \times \frac{\vec{B}}{\mu_0} - \nabla \times \vec{M} = \vec{J}_f$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_f$$

<u>Define</u>

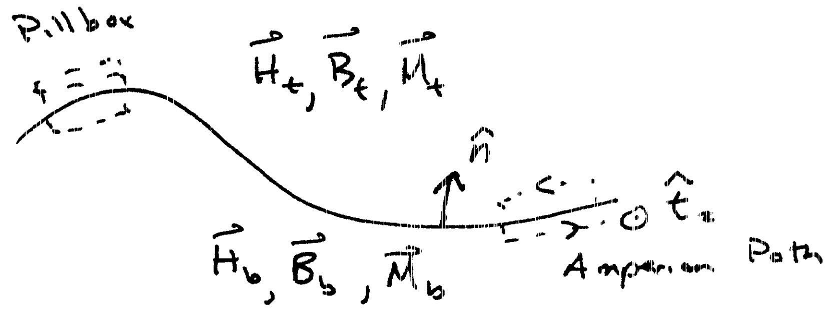
$$\vec{H} \equiv \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\mu_0 \vec{H} = \vec{B} - \mu_0 \vec{M}$$

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$$\nabla \times \vec{H} = \vec{J}_f \quad \Rightarrow \quad \oint_C \vec{H}\cdot d\vec{l} = I_{f\,enc}$$

Boundary Conditions



Gaussian Pillbox

$$\int \nabla\cdot B \, da = 0 = \mu_0 \int \vec{H}\cdot d\vec{a} + \mu_0 \int \vec{M}\cdot d\vec{a}$$

$$H_t^{\perp} - H_b^{\perp} = -\left(M_t^{\perp} - M_b^{\perp}\right)$$

$$H_t = \vec{H}_t \cdot \hat{n}$$

Amperian Path

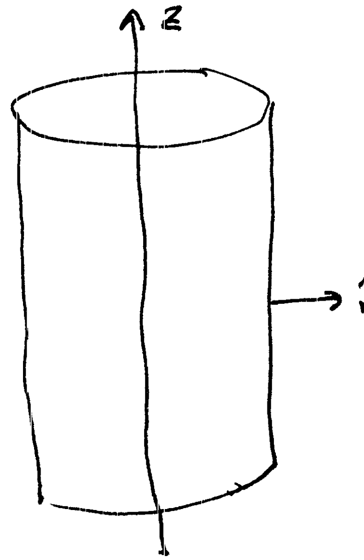
$$\oint_C \vec{H}\cdot d\vec{l} = I_{f\,enc}$$

$$\cancel{\vec{H}_t\cdot t} \quad \left| H_t^{\parallel} - H_b^{\parallel} \right| = \left| K_b \cdot \hat{t} \right|$$

To remove absolute values, I have to be careful of directions. In general,

$$\vec{H}_t^{\parallel} - \vec{H}_b^{\parallel} = \vec{K}_f \times \hat{n}$$

Ex Return to infinite cylinder with magnetization $\vec{M}$.



S/n There is no free current, so

$$\nabla \times \vec{H} = 0$$

This suggests a solution $\vec{H} = 0$, if BC can be met.

$$\underset{\overset{\|}{0}}{H_o^\perp - H^\perp} = \underset{\overset{\|}{0}}{-\left(M_2^\perp - M_1^\perp\right)}$$

since $\vec{M} = M_0 \hat{z}$

$\vec{M} \cdot \hat{s} = 0$

$$\underset{\overset{\|}{0}}{\vec{H}_t^\parallel - \vec{H}_b^\parallel} = \underset{\overset{\|}{0}}{\vec{K}_f} \times \hat{n}$$

So $\vec{H}=0$ everywhere. Note, for a finite cylinder the first B.C. fails on the top and bottom.

<u>Outside</u>

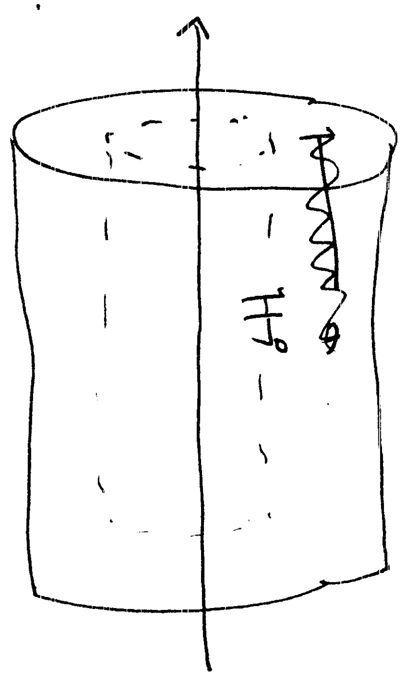
$$\mu_0 \vec{H}_o = \vec{B}_o - \mu_0 \vec{M} = \vec{B}_o = 0$$

<u>Inside</u>

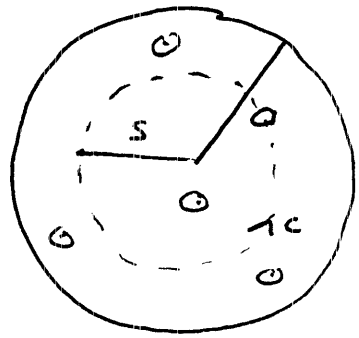
$$\mu_0 \vec{H}_i = \vec{B}_i - \mu_0 \vec{M} = 0$$

$$\vec{B}_i = \mu_0 \vec{M} = \mu_0 M_o \hat{z} \quad \checkmark$$

<u>Ex</u> A conductor of radius a carries a current $\vec{J}_f = \gamma s^2 \hat{z}$. Compute what you can.



End View



Positive normal out of part by RHR

Inside The Conductor

$$I_{fenc} = \int_0^s J_f \, da = \int_0^{2\pi} d\phi \int_0^s ds \, s \, J_f$$

$$= 2\pi \int_0^s ds \, \gamma s^3 = \frac{2\pi \gamma s^4}{4}$$

$$\oint_C \vec{H} \cdot d\vec{l} = I_{fenc} = 2\pi s H$$

$$\vec{H}_i = \frac{I_{fenc}}{2\pi s} \text{ ccw}$$

$$\vec{H}_i = \frac{2\pi \gamma s^4}{8\pi s} \text{ ccw} = \frac{\gamma s^3}{4} \text{ ccw}$$

Outside Conductor

$$I_{fenc} = \frac{\pi \gamma a^4}{2}$$

$$\vec{H}_o = \frac{\pi \gamma a^4 / 2}{2\pi s} \text{ ccw} = \frac{\gamma a^4}{4s} \text{ ccw}$$

Outside the Conductor $\vec{M} = 0$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu_0}$$

$$\vec{B}_0 = \mu_0 \vec{H}_0 = \frac{\mu_0 \gamma a^4}{4s} \quad \text{ccw}$$

Inside the Conductor the magnetization is unknown so we cannot find $\vec{B}$.