Homework 4

Due Monday 3/1/2010 - at 5:00pm

Reading Assignment - Chapter 4 and Section 3.4

Griffiths' Problems

3.28

4.5

4.10

4.15

~~**4.16**~~

4.18

4.20

4.21

4.24

4.26

4.36

3.28

$$\vec{P} = \int \vec{r}\, \sigma(\vec{r})\, da \qquad da = R\, d\theta\, R \sin\theta\, d\phi$$

$P_x = P_y = 0$ by symmetry

$$P_z = \int z\, \sigma(\vec{r})\, R\, d\theta\, R \sin\theta\, d\phi$$

$$z = r\cos\theta = R\cos\theta$$

$$P_z = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta\, \underbrace{(R\cos\theta)}_{z}\, \underbrace{(k\cos\theta)}_{\sigma}\, \underbrace{R^2 \sin\theta}_{da}$$

$$P_z = 2\pi k R^3 \int_0^{\pi} \cos^2\theta \sin\theta\, d\theta$$

Let $u = \cos\theta \qquad du = -\sin\theta\, d\theta$

$$P_z = -2\pi k R^3 \int_1^{-1} u^2\, du = 2\pi k R^3 \left. \frac{u^3}{3} \right|_{-1}^{1}$$

$$= \frac{4}{3} \pi k R^3$$

$$\vec{p} = \frac{4}{3}\pi k R^3 \hat{z}$$

(b) The dipole potential is

$$V_{dip} = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2}$$

$$= \frac{|p|\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\frac{4}{3}\pi k R^3 \cos\theta}{4\pi\varepsilon_0 r^2}$$

$$= \frac{kR^3}{3\varepsilon_0 r^2}\cos\theta$$

which is the exact potential, so there are no higher order multipoles.

4.5 Torque on dipole $\vec{\tau} = \vec{p} \times \vec{E}$

Electric field of dipole, $\vec{p} = p\hat{y}$, along axis.

<u>x-axis</u> $\vec{E} = -\frac{kp}{x^3}\hat{y}$ <u>y-axis</u> $\vec{E} = \frac{2kp}{y^3}\hat{y}$

<u>Torque on p_1 due to p_2</u>

$$\vec{E}_{21} = \frac{2kp_2}{r^3}\hat{x} \qquad \vec{p}_1 = p_1\hat{y}$$

$$\vec{\tau}_{21} = \vec{p}_1 \times \vec{E}_{21} = (p_1\hat{y}) \times \left(\frac{2kp_2}{r^3}\hat{x}\right)$$

$$= -\frac{2kp_1p_2}{r^3}\hat{z}$$

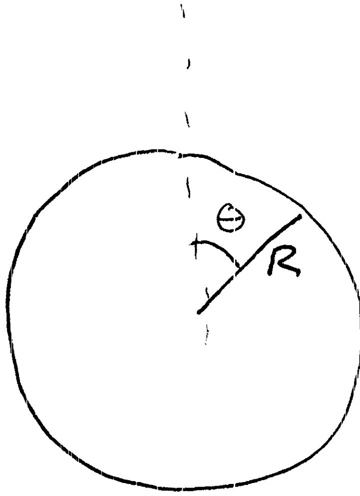
<u>Torque on p_2 due to p_1</u>

$$\vec{E}_{12} = -\frac{kp_1}{r^3}\hat{y} \qquad \vec{p}_2 = p_2\hat{x}$$

$$\vec{\tau}_{12} = \vec{p}_2 \times \vec{E}_{12} = (p_2\hat{x}) \times \left(-\frac{kp_1}{r^3}\hat{y}\right)$$

$$= -\frac{kp_1p_2}{r^3}\hat{z}$$

4.10



(a)

$$\rho_b = -\nabla \cdot \vec{P} = -\nabla \cdot k(x, y, z)$$

$$= -3k$$

$$\sigma_b = \vec{P} \cdot \hat{n} = \vec{P} \cdot \hat{r} = (kR\hat{r}) \cdot \hat{r} = kR$$

(b) We have a system with a uniform surface charge $\sigma = kR$ and a uniform volume charge $\rho = -3k$. Apply Gauss Law.

Inside Sphere $r < R$

$$Q_{enc} = \frac{4}{3}\pi r^3 \rho = -\frac{4}{3}\pi 3k r^3 = -4\pi k r^3$$

Gauss $\Phi = 4\pi r^2 E = \frac{Q_{enc}}{\varepsilon_0}$

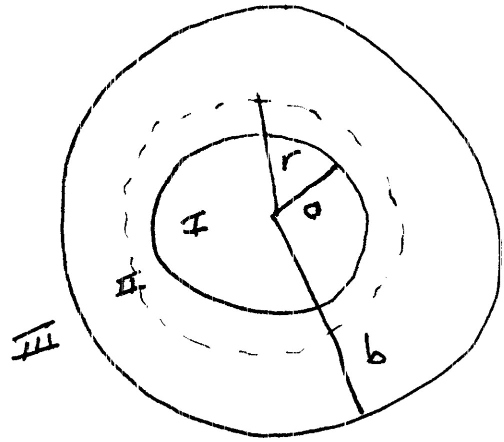
$$\vec{E} = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{-4/3\,\pi 3k r^3}{4\pi\varepsilon_0 r^2}\hat{r} = -\frac{kr}{\varepsilon_0}\hat{r}$$

Outside Sphere $\quad Q_{enc} = \left(\frac{4}{3}\pi R^3\right)\underbrace{(-3k)}_{\rho} + \left(4\pi R^2\right)\underbrace{(kR)}_{\sigma}$

$$Q_{enc} = -4\pi k R^3 + 4\pi k R^3 = 0$$

$$\vec{E} = 0$$

4.15



$$\vec{P} = \frac{k}{r}\hat{r}$$

(a) Bound charge

$$\sigma_b = \vec{P}\cdot\hat{n}$$

Inner Surface $\hat{n} = -\hat{r}$

$$\sigma_b = -\frac{k}{a} \qquad Q_{inner} = 4\pi a^2 \sigma_b = -4\pi k a$$

Outer Surface $\hat{n} = \hat{r}$

$$\sigma_b = \frac{k}{b} \qquad Q_{outer} = 4\pi b^2 \sigma_b = 4\pi k b$$

Volume Charge

$$\rho_b = -\nabla\cdot\vec{P} = -k\,\nabla\cdot\left(\frac{\hat{r}}{r}\right)$$

$$= \frac{-k}{r^2}\frac{\partial}{\partial r}\left(r^2\cdot\frac{1}{r}\right) = -\frac{k}{r^2}$$

Region III $\quad Q_{enc} = Q_{inner} + Q_{vol}(b) + Q_{outer}$

$$= -4\pi k a - 4\pi k(b-a) + 4\pi k b$$

$$= 0$$

$$\vec{E}_{III} = 0$$

(b) There is no free charge,

$$\nabla \cdot \vec{D} = 0 \quad \Rightarrow \quad \vec{D} = 0$$

(constant solution violates spherical symmetry)

In region I, III $\quad \vec{P} = 0$

$$\vec{D} = 0 = \varepsilon_0 \vec{E} + \vec{P} \Rightarrow \vec{E} = 0$$

$$\vec{E}_I = 0 \qquad \vec{E}_{III} = 0$$

In region II,

$$\vec{D} = 0 = \varepsilon_0 \vec{E}_{II} + \vec{P}$$

$$\vec{E}_{II} = -\frac{\vec{P}}{\varepsilon_0} = -\frac{k}{\varepsilon_0 r}\hat{r}$$

Gauss Law

Region I $\quad r < a \qquad Q_{enc} = 0 \qquad \vec{E}_I = 0$

Region II $\quad a < r < b \qquad Q_{enc} = Q_{inner} + Q_{vol}(r)$

$$Q_{vol}(r) = \int_a^r 4\pi r^2 \rho_0 \, dr$$

$$= \int_a^r 4\pi r^2 \left(-\frac{k}{r^2} \right) dr$$

$$= -4\pi k (r - a)$$

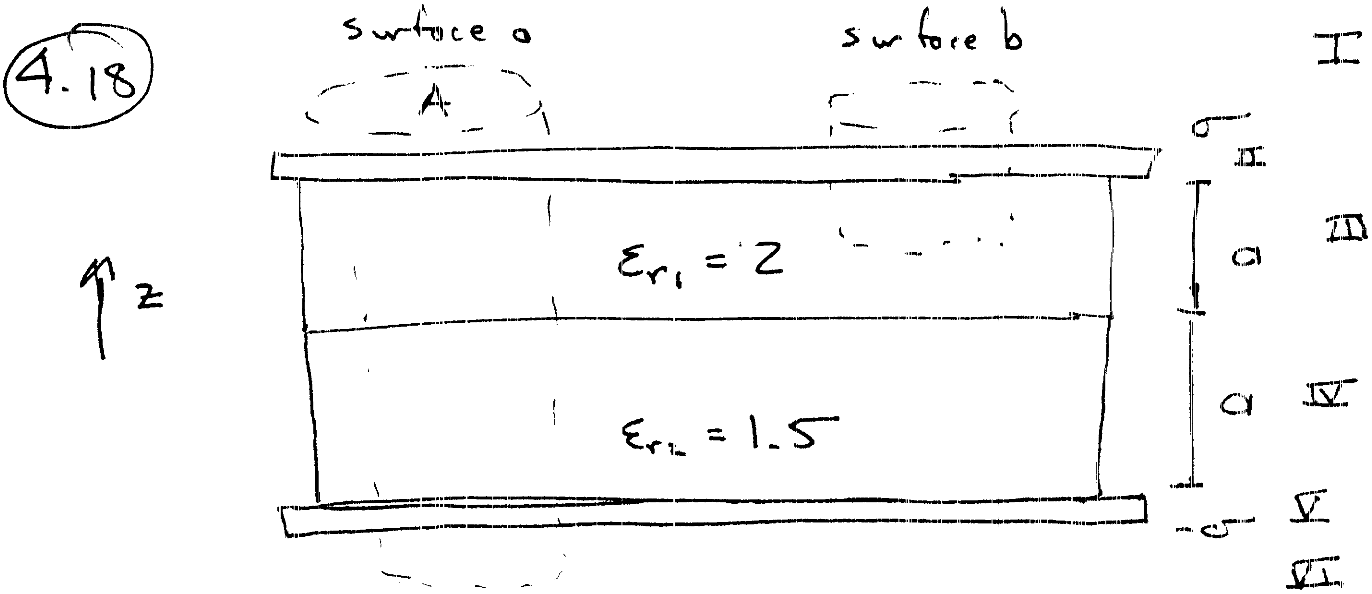
$$Q_{enc} = -4\pi k a - 4\pi k (r - a) = -4\pi k r$$

Gauss $\quad \Phi = 4\pi r^2 E = Q_{enc}/\varepsilon_0$

$$\vec{E}_{II} = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2} \hat{r} = \frac{-4\pi k r}{4\pi\varepsilon_0 r^2} \hat{r}$$

$$\vec{E}_{II} = \frac{-k}{\varepsilon_0 r} \hat{r}$$

4.18



(a) The total free charge in surface a is

$$Q_{fenc} = \sigma A + (-\sigma A) = 0$$

by symmetry $\vec{D}_{I} = \vec{D}_{VI} = 0$

The free charge in surface b is $Q_{fenc} = \sigma A$

Apply Gauss Law

$$\Phi = D_I A - D_{III} A = Q_{fenc}$$

$$\vec{D}_{III} = -\sigma \hat{z} = \vec{D}_{IV}$$

Naturally, $\vec{D}_{II}$ and $\vec{D}_{V} = 0$

(b) $\vec{D}_{III} = \varepsilon_{r1}\varepsilon_0 \vec{E}_{III}$

$$\vec{E}_{III} = \frac{-\sigma}{\varepsilon_{r1}\varepsilon_0 \cancel{\varepsilon_0}}\hat{z} = \frac{-\sigma}{2\varepsilon_0}\hat{z} \quad \text{top slab}$$

$$\vec{E}_{IV} = \frac{-\sigma}{\varepsilon_{r2}\varepsilon_0}\hat{z} = -\frac{2}{3}\frac{\sigma}{\varepsilon_0}\hat{z} \quad \text{bottom slab}$$

(c) Polarization

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

Top Slab $\quad \chi_{e1} = \varepsilon_{r1} - 1 = 1$

$$\vec{P}_{III} = \varepsilon_0 \chi_{e1} \vec{E}_{III} = -\frac{\chi_{e1}\sigma\varepsilon_0}{\varepsilon_{r1}\varepsilon_0}\hat{z}$$

$$= -\frac{\chi_{e1}}{\varepsilon_{r1}}\sigma\hat{z} = -\frac{\chi_{e1}}{(\chi_{e1}+1)}\sigma\hat{z}$$

$$= -\frac{1}{2}\sigma\hat{z}$$

$$\chi_{e2} = 3/2 - 1 = 1/2$$

Bottom Slab

$$\vec{P}_{IV} = \frac{-\chi_{e2}}{\varepsilon_{r2}}\sigma\hat{z} = \frac{-1/2}{3/2}\sigma\hat{z}$$

$$= \frac{-1}{3}\sigma\hat{z}$$

(d) $|\Delta V| = E_{III}\, a + E_{IV}\, a$

$$= \frac{\sigma}{2\varepsilon_0} a + \frac{2\sigma}{3\varepsilon_0} a$$

$$= \frac{7}{6}\frac{\sigma}{\varepsilon_0} a$$

(e) Bound charge –

<u>Slab 1</u> top – $\hat{n} = \hat{z}$

$$\sigma_{1,top} = \vec{P}\cdot\hat{z} = -\frac{1}{2}\sigma$$

bottom – $\hat{n} = -\hat{z}$

$$\sigma_{1,bottom} = \frac{1}{2}\sigma$$

<u>Slab 2</u> top $\hat{n} = \hat{z}$

$$\sigma_{2,top} = \vec{P}\cdot\hat{z} = -\frac{1}{3}\sigma$$

$$\sigma_{2,bottom} = \vec{P}\cdot(-\hat{z}) = \frac{1}{3}\sigma$$

(f) Field in top slab, superposition of

σ, $-\sigma$ and $-\frac{1}{2}\sigma$, $\frac{1}{2}\sigma$

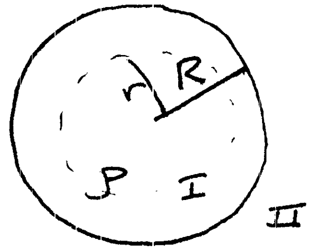
$$\vec{E}_{III} = -\frac{\sigma}{\varepsilon_0}\hat{z} + \frac{\frac{1}{2}\sigma}{\varepsilon_0}\hat{z} = -\frac{1}{2}\frac{\sigma}{\varepsilon_0}\hat{z}$$

Field in bottom slab superposition of

$\sigma, -\sigma$ and $-\frac{1}{3}\sigma, \frac{1}{3}\sigma$

$$\vec{E} = -\frac{\sigma}{\varepsilon_0}\hat{z} + \frac{\frac{1}{3}\sigma}{\varepsilon_0}\hat{z} = -\frac{2}{3}\frac{\sigma}{\varepsilon_0}\hat{z}$$

4.20



Gauss Law

Region I $\quad r < R$

$$Q_{enc} = \frac{4}{3}\pi r^3 \rho$$

$$\Phi = 4\pi r^2 D = Q_{enc}$$

$$\vec{D}_I = \frac{\frac{4}{3}\pi r^3 \rho}{4\pi r^2}\hat{r} = \frac{\rho r}{3}\hat{r}$$

$$\vec{D}_I = \varepsilon_r \varepsilon_0 \vec{E}_I$$

$$\vec{E}_I = \frac{\rho r}{3\varepsilon_r \varepsilon_0}\hat{r} = \frac{\rho r}{3\varepsilon_r \varepsilon_0}\hat{r}$$

Region II $\quad r > R$

$$Q_{fenc} = \frac{4}{3}\pi R^3 \rho$$

$$\vec{D}_{II} = \frac{Q_{fenc}}{4\pi r^2}\hat{r} = \frac{\frac{4}{3}\pi R^3 \rho}{4\pi r^2}\hat{r}$$

$$= \frac{R^3 \rho}{3 r^2}\hat{r}$$

$$\vec{D}_{II} = \varepsilon_0 \vec{E}_{II}$$

$$\vec{E}_{II} = \frac{R^3 \rho}{3\varepsilon_0 r^2}\hat{r}$$

Potential Difference

$$\Delta V_{\infty 0} = -\int_{\infty}^{0} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{R} \vec{E}_{II}\cdot d\vec{l} - \int_{R}^{0} \vec{E}_{I}\cdot d\vec{l}$$

$$d\vec{l} = dr\,\hat{r} \qquad (dr<0 \text{ because of limits})$$

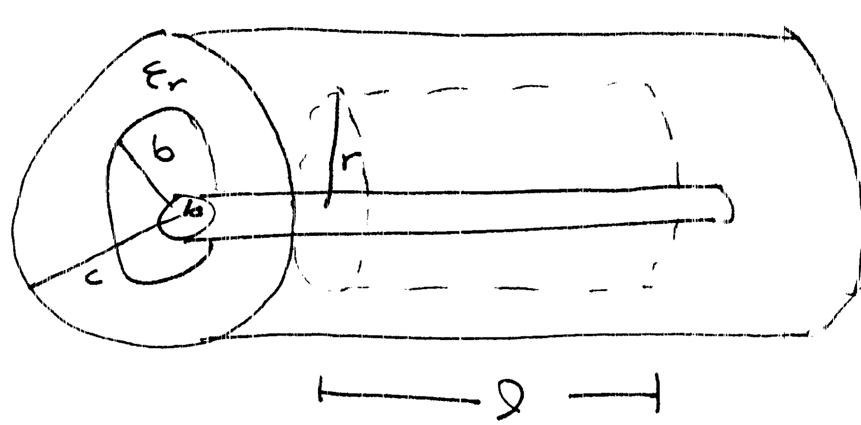
$$\Delta V_{\infty 0} = -\int_{\infty}^{R} \frac{R^3 \rho}{3\varepsilon_0 r^2}\,dr - \int_{R}^{0} \frac{\rho r}{3\varepsilon_r \varepsilon_0}\,dr$$

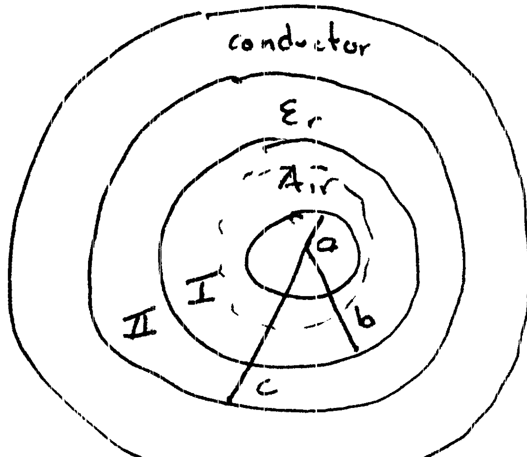
$$= \left.\frac{R^3 \rho}{3\varepsilon_0 r}\right|_{\infty}^{R} - \left.\frac{\rho r^2}{6\varepsilon_r \varepsilon_0}\right|_{R}^{0}$$

$$\Delta V_{\infty 0} = \frac{R^3 \rho}{3 \varepsilon_0} \left(\frac{1}{R} - \frac{1}{\infty} \right) + \frac{\rho R^2}{6 \varepsilon_0 \varepsilon_r}$$

$$= \frac{\rho R^2}{3 \varepsilon_0} \left(1 + \frac{1}{2 \varepsilon_r} \right)$$

4.21





Add +Q charge per length l of the Gaussian surface.

Q_{fenc} in cylindrical Gaussian surface of radius r

is $Q_{fenc} = Q$

<u>Gauss Law</u> $\Phi_d = 2\pi r l D = Q_{fenc}$

$$\vec{D} = \frac{Q}{l} \cdot \frac{1}{2\pi r}$$

Region I $\quad a < r < b$

$$\vec{D} = \varepsilon_0 \vec{E}_I$$

$$\vec{E}_I = \frac{Q}{\ell} \cdot \frac{1}{2\pi\varepsilon_0 r} \hat{r}$$

Note, using $\hat{r} = \hat{s}$ because of habit. Definitely a cylindrical problem.

Region II $\quad b < r < c$

$$\vec{D} = \varepsilon_r \varepsilon_0 \vec{E}_{II}$$

$$\vec{E}_{II} = \frac{Q}{\ell} \frac{1}{2\pi\varepsilon_0 \varepsilon_r r} \hat{r}$$

Potential Difference

$$|\Delta V| = |\Delta V_I| + |\Delta V_{II}| \quad \text{(fields in same direction)}$$

$$|\Delta V_I| = \left| -\int_a^b \vec{E}_I \cdot d\vec{\ell} \right| \qquad d\vec{\ell} = \hat{r}\, dr$$

$$= \left| -\frac{Q}{\ell} \cdot \frac{1}{2\pi\varepsilon_0} \int_a^b \frac{dr}{r} \right| = \frac{Q}{\ell} \cdot \frac{1}{2\pi\varepsilon_0} \ln(b/a)$$

$$|\Delta V_{II}| = \left| -\int_b^c \vec{E}_{II} \cdot d\vec{l} \right| = \left| -\frac{Q}{l} \frac{1}{2\pi\varepsilon_0\varepsilon_r} \int_b^c \frac{dr}{r} \right|$$

$$= \frac{Q}{l} \frac{1}{2\pi\varepsilon_0\varepsilon_r} \ln(c/b)$$

$$|\Delta V| = |\Delta V_I| + |\Delta V_{II}|$$

$$= \frac{Q}{2\pi\varepsilon_0 l} \left(\ln(b/a) + \frac{1}{\varepsilon_r} \ln(c/b) \right)$$

$$C = \frac{Q}{|\Delta V|} = \frac{2\pi\varepsilon_0 l}{\ln(b/a) + \frac{1}{\varepsilon_r} \ln(c/b)}$$

$$\frac{C}{l} = \frac{2\pi\varepsilon_0}{\ln(b/a) + \frac{1}{\varepsilon_r} \ln(c/b)}$$

4.24 Let the surface of the sphere be at zero potential.

Solutions to Laplace's Eqn

Conductor
$V_c = 0 \qquad r < a$

Dielectric
$$V_K = \sum A_n r^n P_n(\cos\theta) + B_n r^{-(n+1)} P_n(\cos\theta)$$

cannot throw away terms because r does not go to zero ~~in the~~ or ∞ in the dielectric. $(a < r < b)$

Outside
$$V_o = \sum C_n r^{-(n+1)} P_n(\cos\theta) - E_0 r P_1(\cos\theta)$$

discard r^n terms because they blow up except $V = -E_0 r\cos\theta$
$\Rightarrow \vec{E} = E_0 \hat{z}$

Boundary Conditions

$$V_c(a,\theta) = 0 = V_K(a,0)$$

V_K must be zero term by term

$$A_n a^n + B_n a^{-(n+1)} = 0$$

$$B_n = -A_n a^{2n+1}$$

$$V_K = \sum A_n P_n(\cos\theta)\left(r^n - \frac{a^{2n+1}}{r^{n+1}}\right)$$

<u>Continuous at $r=b$</u>

$$V_K(b,\theta) = V_0(b,\theta)$$

$$\sum A_n P_n(\cos\theta)\left(b^n - \frac{a^{2n+1}}{b^{n+1}}\right)$$

$$= \sum C_n \frac{1}{b^{n+1}} P_n(\cos\theta) - E_0 b P_1$$

<u>$n=1$</u> $$A_1\left(b - \frac{a^3}{b^2}\right) = \frac{C_1}{b^2} - E_0 b$$

<u>$n\neq 1$</u> $$A_n\left(b^n - \frac{a^{2n+1}}{b^{n+1}}\right) = \frac{C_n}{b^{n+1}}$$

Electrostatic B.C. at $r=b$

$$\left.\frac{\partial V_0}{\partial r}\right|_b - \varepsilon_r \left.\frac{\partial V_k}{\partial r}\right|_b = -\sigma_f = 0$$

$$\sum -(n+1) C_n \frac{1}{b^{n+2}} P_n(\cos\theta) - E_0 P_1(\cos\theta)$$

$$- \varepsilon_r \sum A_n P_n(\cos\theta)\left(n b^{n-1} + (n+1)\frac{a^{2n+1}}{b^{n+2}} \right)$$

$$= 0$$

<u>$n=1$</u>

$$-2\frac{C_1}{b^3} - E_0 - \varepsilon_r A_1\left(1 + 2\frac{a^3}{b^3}\right) = 0$$

<u>$n \neq 1$</u>

$$-(n+1)\frac{C_n}{b^{n+2}} - \varepsilon_r A_n\left(n b^{n-1} + (n+1)\frac{a^{2n+1}}{b^{n+2}} \right)$$

$$= 0$$

For $n \neq 1$, we have two linear equations in two unknowns,

$$c_1 A_n + c_2 C_n = 0$$

$$c_3 A_1 + c_4 C_n = 0$$

the solution is $A_n = B_n = C_n = 0$

Solve $n = 1$

$$-2C_1 - b^3 E_0 - \varepsilon_r A_1 (b^3 + 2a^3) = 0$$

$$A_1 (b^3 - a^3) = C_1 - E_0 b^3$$

$$C_1 = A_1 (b^3 - a^3) + E_0 b^3$$

$$-2\left(A_1 (b^3 - a^3) + E_0 b^3\right) - b^3 E_0 - \varepsilon_r A_1 (b^3 + 2a^3) = 0$$

$$-A_1\left(+2(b^3 - a^3) + \varepsilon_r (b^3 + 2a^3)\right) = 3b^3 E_0$$

$$A_1 = \frac{-3b^3 E_0}{2(b^3 - a^3) + \varepsilon_r (b^3 + 2a^3)}$$

$$A_1 = \frac{-3E_0}{2\left(1 - a^3/b^3\right) + \varepsilon_r\left(1 + \frac{2a^3}{b^3}\right)}$$

<u>Potential in Dielectric</u>

$$V_R = P_1(\cos\theta)\, A_1 \left(r^1 - \frac{a^3}{r^2}\right)$$

$$= \frac{-3E_0 \cos\theta}{2\left(1 - a^3/b^3\right) + \varepsilon_r\left(1 + \frac{2a^3}{b^3}\right)} \left(r - \frac{a^3}{r^2}\right)$$

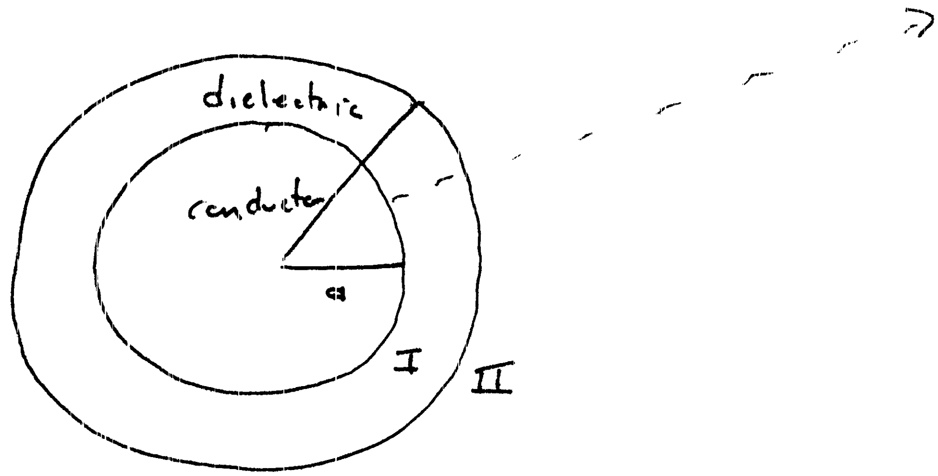
<u>Field in Dielectric</u>

$$\vec{E} = -\nabla V =$$

$$\frac{+3E_0}{2\left(1 - \frac{a^3}{b^3}\right) + \varepsilon_r\left(1 + \frac{2a^3}{b^3}\right)} \left[\left(1 + \frac{2a^3}{r^3}\right)\cos\theta\, \hat{r} - \left(1 - \frac{a^3}{r^3}\right)\sin\theta\, \hat{\theta}\right]$$

using gradient from front cover.

4.26



Electric field

$$\vec{E}_I = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r r^2}\hat{r} \qquad \vec{E}_{II} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Potential Difference $\Delta V_{a\infty}$

$$\Delta V = -\int_a^b \vec{E}_I \cdot d\vec{l} - \int_b^\infty \vec{E}_{II} \cdot d\vec{l}$$

$$= -\int_a^b \frac{Q}{4\pi\varepsilon_0 \varepsilon_r r^2} dr - \int_b^\infty \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$= \frac{Q}{4\pi\varepsilon_0 \varepsilon_r}\left(\frac{1}{b} - \frac{1}{a}\right) - \frac{Q}{4\pi\varepsilon_0 b}$$

Capacitance

$$C = \frac{Q}{|\Delta V|} = \frac{4\pi\varepsilon_0}{\frac{1}{\varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{1}{b}}$$

$$= \frac{4\pi\varepsilon_0\varepsilon_r}{\frac{1}{a} - \frac{1}{b} + \frac{\varepsilon_r}{b}}$$

$$= \frac{4\pi\varepsilon_0\varepsilon_r}{\frac{b - a + \varepsilon_r a}{ab}} = \frac{4\pi\varepsilon_0\varepsilon_r ab}{b + (\varepsilon_r - 1)a}$$

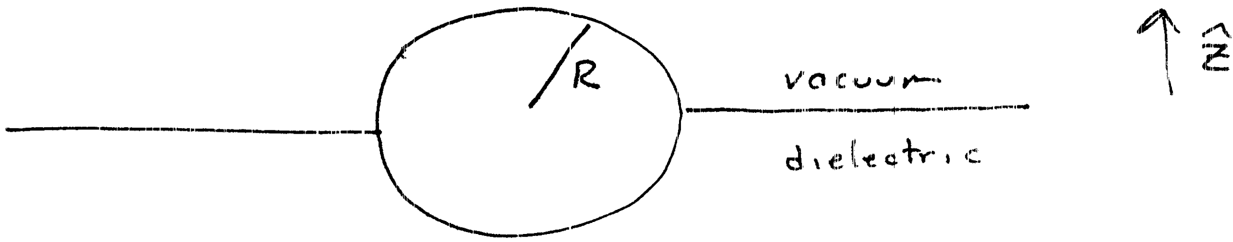
$$= \frac{4\pi\varepsilon_0\varepsilon_r ab}{b - \chi_e a}$$

Energy

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Q^2\left(\frac{b - \chi_e a}{4\pi\varepsilon_0\varepsilon_r ab}\right)$$

$$= \frac{Q^2}{8\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{a} - \frac{\chi_e}{b}\right)$$

4.36



(a) $V = \dfrac{V_0 R}{r}$ if potential same as missing dielectric.

Field everywhere

$$\vec{E} = -\nabla V = \frac{V_0 R}{r^2}\,\hat{r}$$

Polarization $\vec{P} = 0, \quad z > 0$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} = \frac{\varepsilon_0 V_0 R \chi_e}{r^2}\,\hat{r} \qquad z < 0$$

Bound Charge

$$\rho_b = -\nabla \cdot \vec{P} = 0$$

$$\sigma_b = \vec{P} \cdot \hat{z} = 0 \quad \text{interface between planes}$$

$$\sigma_b = -\vec{P} \cdot \hat{n}\Big|_R = -\frac{\varepsilon_0 V_0 \chi_e}{R} \quad \text{at surface of sphere.}$$

- for inward normal

Free Charge

$z > 0$ Using Gaussian Pillbox

$$\Phi = \vec{E}_c(R) \cdot \hat{n} A - 0 = \frac{\sigma_{+c} A}{\varepsilon_0}$$

$$\sigma_{c+} = \varepsilon_0 \vec{E}_0(R) = \frac{\varepsilon_0 V_0}{R}$$

$z < 0$ Pillbox encloses σ_{c-} and σ_b

$$\Phi = \vec{E}_k(R) \cdot \hat{r} A - 0 = \frac{(\sigma_{c-} + \sigma_b) A}{\varepsilon_0}$$

$$\varepsilon_0 \left(\frac{V_0 R}{R^2} \right) = \sigma_{c-} + \sigma_b$$

$$\sigma_{c-} = \frac{\varepsilon_0 V_0}{R} + \frac{\varepsilon_0 V_0 \chi_e}{R}$$

$$= \frac{\varepsilon_0 V_0}{R} \varepsilon_r$$

(b) The total charge is σ_{c+} over the entire sphere because $\sigma_{c+} = \sigma_{c-} + \sigma_b$, which does produce the field.

(c) Since we meet the boundary conditions, we must have the solution.

(d) (b) yes, (a) no. We would not meet the electrostatic boundary condition at the boundary.