Homework 7

Due Monday 4/12/2010 - at 5:00pm

Reading Assignment - Chapter 7

Griffiths' Problems

7.1(a) and (b)

7.4

7.8

7.11

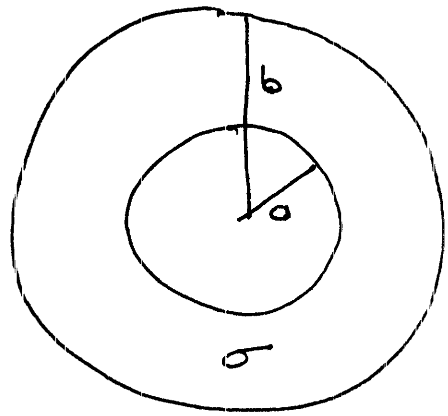
7.12

7.13

7.15

7.16

7.1



Assume a current I flows from a → b.

The current density through a surface of radius r is

$$J = \frac{I}{4\pi r^2}$$

The electric field at a radius r is then given by Ohm's law

$$E = \frac{J}{\sigma} = \frac{I}{4\pi r^2 \sigma}$$

The potential difference between the shells is

$$\Delta V = -\int_{a \to b} \vec{E} \cdot d\vec{\ell} = \frac{I}{4\pi\sigma} \int_a^b \frac{dr}{r^2}$$

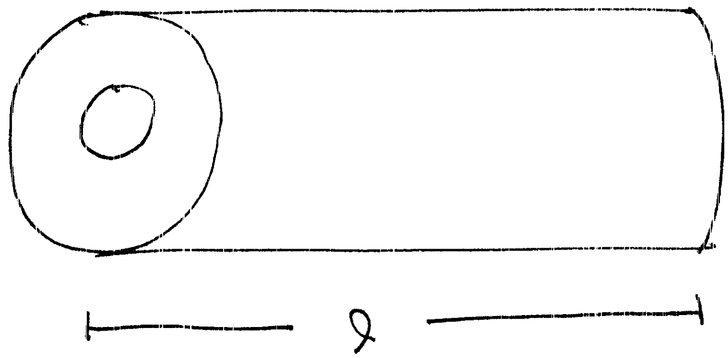
$$|\Delta V| = \frac{I}{4\pi\sigma}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{I}{4\pi\sigma a b}(b-a)$$

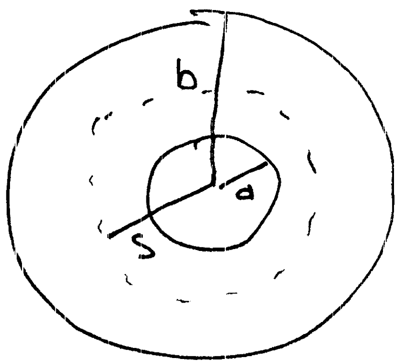
$$I = \frac{4\pi\sigma a b V}{b - a}$$

(b) The resistance is

$$R = \frac{V}{I} = \frac{b - a}{4\pi\sigma a b}$$

7.4





$$\sigma = \frac{K}{s}$$

Let the current flowing between the cylinders be I

The current density flowing through a surface of radius s is then

$$J = \frac{I}{2\pi s \ell}$$

The electric field between the cylinders is

$$E = \frac{J}{\sigma} = \frac{I}{2\pi s \ell \sigma} = \frac{I}{2\pi K \ell}$$

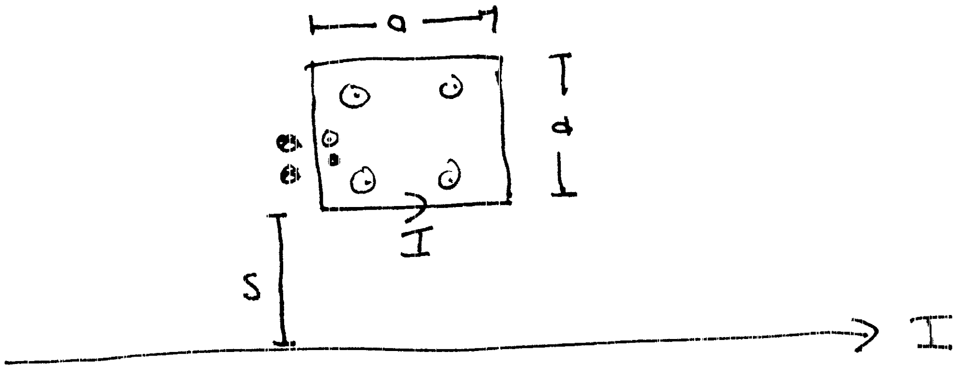
Since the field is constant, the potential difference is

$$\Delta V = E(b-a) = \frac{b-a}{2\pi k \lambda} I$$

So the resistance is

$$R = \frac{\Delta V}{I} = \frac{b-a}{2\pi k \lambda}$$

7.8



(a) Magnetic flux - Slice loop into strips of area $da = a\,ds$

$$\Phi_m = \int B\,da = \int_s^{s+a} \left(\frac{\mu_0 I}{2\pi s} \right) a\,ds$$

where I have used the field of an infinite wire $B = \mu_0 I / 2\pi s$

$$\Phi_m = \frac{\mu_0 I a}{2\pi} \int_s^{s+a} \frac{ds}{s} = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{s+a}{s}\right)$$

(b) By Lenz' law, the magnetic flux out of the page is decreasing as the loop is pulled away so the induced flux is out of the page to oppose the change and the current flows counterclockwise.

If the loop is pulled away at velocity v,

$$s = s_0 + vt$$

$$\Phi_m = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{s_0 + a + vt}{s_0 + vt}\right)$$

$$\text{emf} = \frac{-d\Phi_m}{dt} = -\frac{\mu_0 I a v}{2\pi}\left[\frac{1}{s_0 + a + vt} - \frac{1}{s_0 + vt}\right]$$

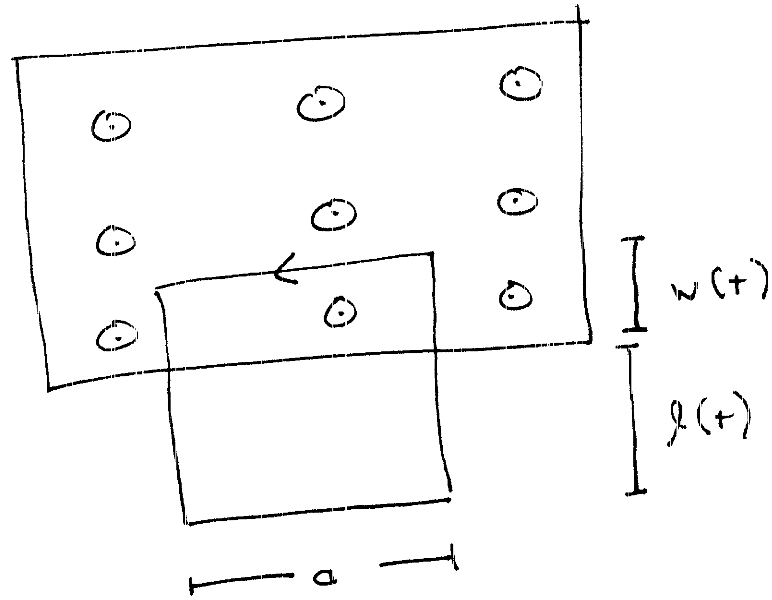
Write this in terms of $s = s_0 + vt$

$$\text{emf} = -\frac{\mu_0 I a v}{2\pi}\left[\frac{1}{s+a} - \frac{1}{s}\right]$$

$$= \frac{\mu_0 I a^2 v}{2\pi s(s+a)}$$

(c) The flux is not changing, so emf = 0.

7.11



The magnetic flux through the loop is

$$\Phi_m = BA = Baw$$

The emf is then

$$\text{emf} = -\frac{d\Phi_m}{dt} = Bav$$

The current is ccw to produce a flux that opposes the change in flux.

The magnetic force on the top wire is

$$\vec{F}_m = I\vec{L}\times\vec{B} = IaB \text{ upward}$$

The current flowing in the loop is

$$I = \frac{\text{emf}}{R}$$

where R is the resistance of the loop.

Therefore the upward magnetic force is

$$F_m = I a B$$

$$= \frac{emf \, a B}{R}$$

$$= \frac{B a v a B}{R} = \frac{v a^2 B^2}{R}$$

The total force on the loop is

$$F = F_m - mg = 0 \quad \text{at terminal velocity}$$

$$\frac{v_t a^2 B^2}{R} = mg$$

$$v_t = \frac{mgR}{a^2 B^2}$$

Now solve for the velocity as a function of time

$$F = m\frac{dv}{dt} = \frac{a^2 B^2}{R} v - mg$$

$$\frac{dv}{dt} = \frac{a^2 B^2}{mR} v - g = g\left(\frac{v}{v_t} - 1\right)$$

$$= \frac{g}{v_t}(v - v_t)$$

Switch coordinate system so downward is positive

$$\frac{dv}{dt} = g - \frac{g}{v_t} v$$

$$= \frac{g}{v_t} (v_t - v)$$

$$\int_0^t \frac{v_t}{g} dt = \int_0^v \frac{dv}{v_t - v}$$

$$\frac{v_t}{g} t = - \ln \left(\frac{v_t - v}{v_t} \right)$$

$$e^{-v_t t/g} = \frac{v_t - v}{v_t}$$

$$v = v_t \left(1 - e^{-v_t t/g} \right)$$

Time for $v = 0.9 v_t$

$$0.9 v_t = v_t \left(1 - e^{-v_t t/g} \right)$$

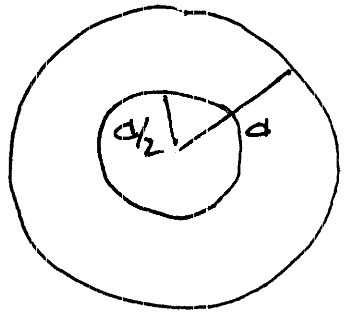
$$0.1 = e^{-v_t t/g}$$

$$\ln(0.1) = -\frac{v_t}{g} t \qquad t = -\frac{g}{v_t} \ln(0.1)$$

I will skip the numerical calculation.

If the loop were cut, there is no induced current, so the loop falls freely.

7.12



The field in the solenoid is given as $B(t) = B_0 \cos \omega t$

The flux through the loop is

$$\Phi_m = BA = \frac{B \pi a^2}{4} = \frac{B_0 \pi a^2}{4} \cos \omega t$$

The emf is by Faraday's law,

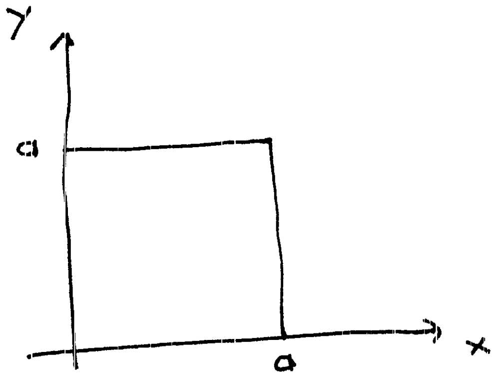
$$\text{emf} = -\frac{d \Phi_m}{dt} = \frac{B_0 \pi a^2 \omega}{4} \cos \omega t$$

and the current

$$I = \frac{\text{emf}}{R} = \frac{B_0 \pi a^2 \omega}{4R} \cos \omega t$$

7.13

$$B = k y^3 t^2$$



$$da = a\,dx$$

<u>Magnetic Flux</u>

$$\Phi_m = \int B\,da = \int_0^a B\,a\,dy$$

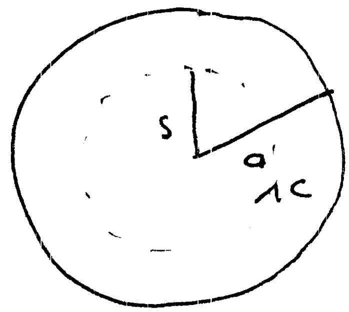
$$= k a t^2 \int_0^a y^3\,dy$$

$$= \frac{k a t^2 a^4}{4} = \frac{k a^5 t^2}{4}$$

emf by Faraday

$$emf = -\frac{d\Phi_m}{dt} = -\frac{k a^5 t}{2}$$

7.15



n turns per length

Magnetic field in solenoid

$$B(t) = \mu_0 n I(t)$$

Magnetic Flux through surface of radius $s < a$

$$\Phi_m = BA = B\pi s^2 = \mu_0 n \pi s^2 I(t)$$

Faraday's Law

$$\text{emf} = \oint \vec{E} \cdot d\vec{l} = 2\pi s E = -\frac{d\Phi_m}{dt} = -\mu_0 n \pi s^2 \frac{dI}{dt}$$

$$E = -\frac{\mu_0 n s}{2} \frac{dI}{dt} \quad \text{ccw}$$

inside solenoid

$$\vec{E} = \frac{-\mu_0 n s}{2} \frac{dI}{dt} \hat{\phi}$$

Outside Solenoid

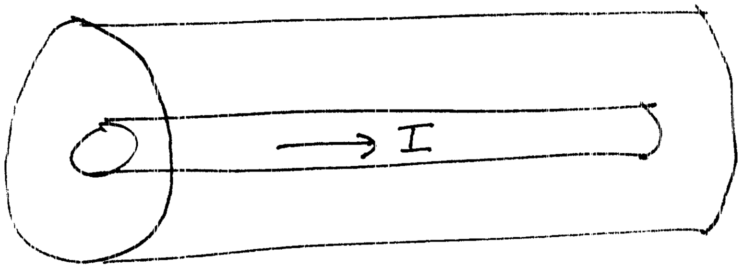
$$\Phi_m = \mu_0 n \pi a^2 I(t)$$

Faraday

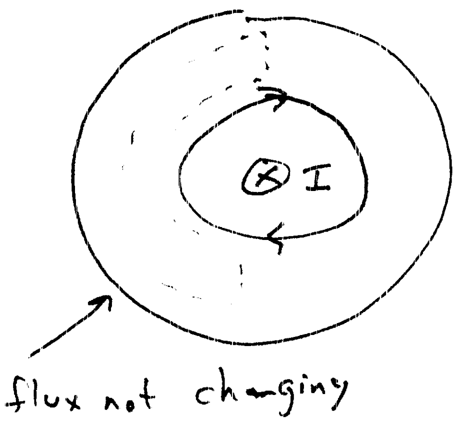
$$2\pi s E = -\frac{d\Phi_m}{dt} = -\mu_0 n \pi a^2 \frac{dI}{dt}$$

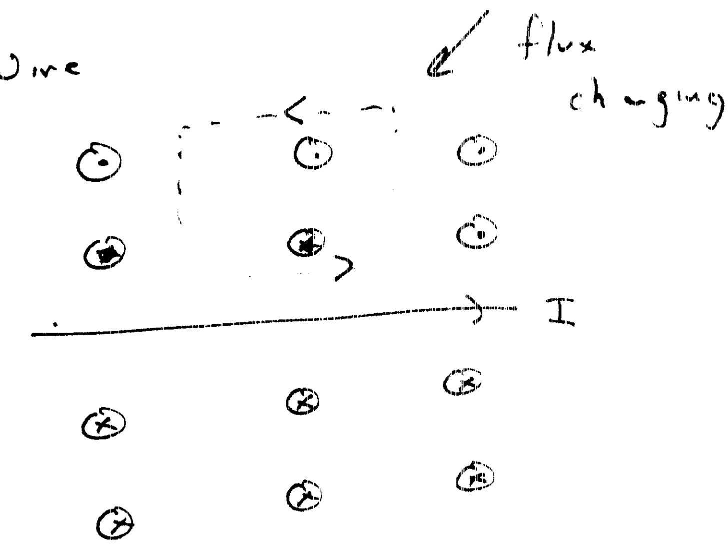
$$\vec{E} = \frac{-\mu_0 n a^2}{2s} \frac{dI}{dt}$$

7.16



Magnetic Field Circles Wire



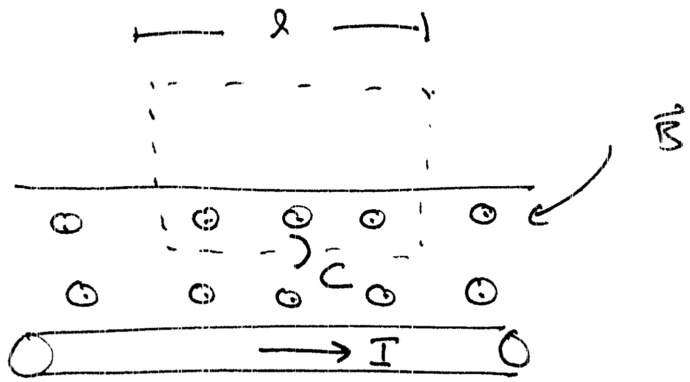


$$\vec{B} = B(t)\,\hat{\phi}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{dB}{dt}\,\hat{\phi} = \left(\frac{\partial E_s}{\partial z} - \frac{\partial E_z}{\partial z}\right)\hat{\phi}$$

$\vec{E}$ cannot be radial ($\vec{E} = E\,\hat{s}$) because no net charge is developed on the wire, therefore the field must point along the axis –

$$\vec{E} = E(t)\,\hat{z}$$



Magnetic Flux through C ~~Field~~

$$\Phi_m = \cancel{BA} \int B\,da \qquad da = l\,ds$$

Magnetic field Ampere's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = 2\pi s B$$

$$B = \frac{\mu_0 I}{2\pi s}$$

Magnetic Flux

$$\Phi_m = \int B\,da = \int_s^a B\,l\,ds$$

$$= \int_s^a \frac{\mu_0 I}{2\pi s}\,l\,ds = \frac{\mu_0 I l}{2\pi}\ln\left(a/s\right)$$

The field outside is zero.

$$\Phi_m = \frac{\mu_0 l I_0}{2\pi} \ln\left(\frac{a}{s}\right) \cos\omega t$$

$$\frac{d\Phi_m}{dt} = -\frac{\mu_0 l I_0 \omega}{2\pi} \sin\omega t \ln\left(a/s\right)$$

<u>Faraday's Law</u>

$$\oint \vec{E}\cdot d\vec{l} = -\overset{0}{E_{outside}}\, l + E_{inside}\, l = -\frac{d\Phi_m}{dt}$$

$$\vec{E}_{inside} = \frac{\mu_0 I_0 \omega}{2\pi} \sin\omega t \ln\left(a/s\right) \hat{z}$$