More Method of Images

The method of images can also be used on spheres and cylinders.

Point Charge Outside Grounded Conducting Sphere

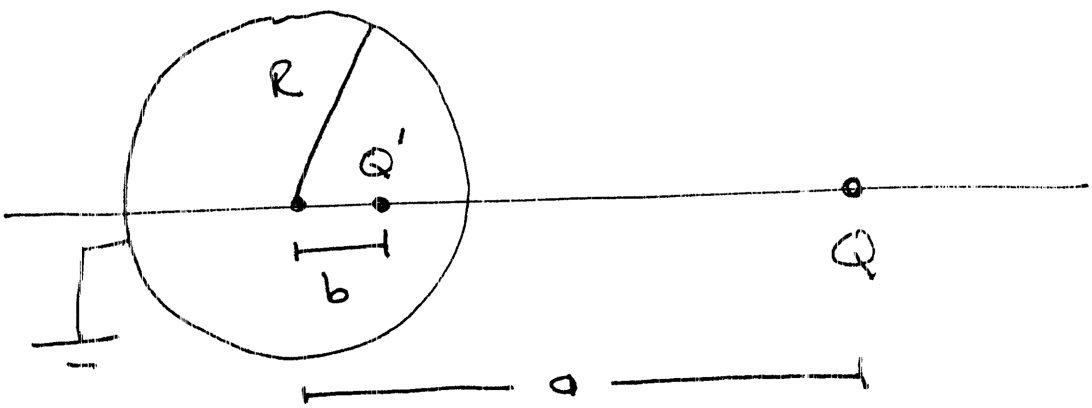
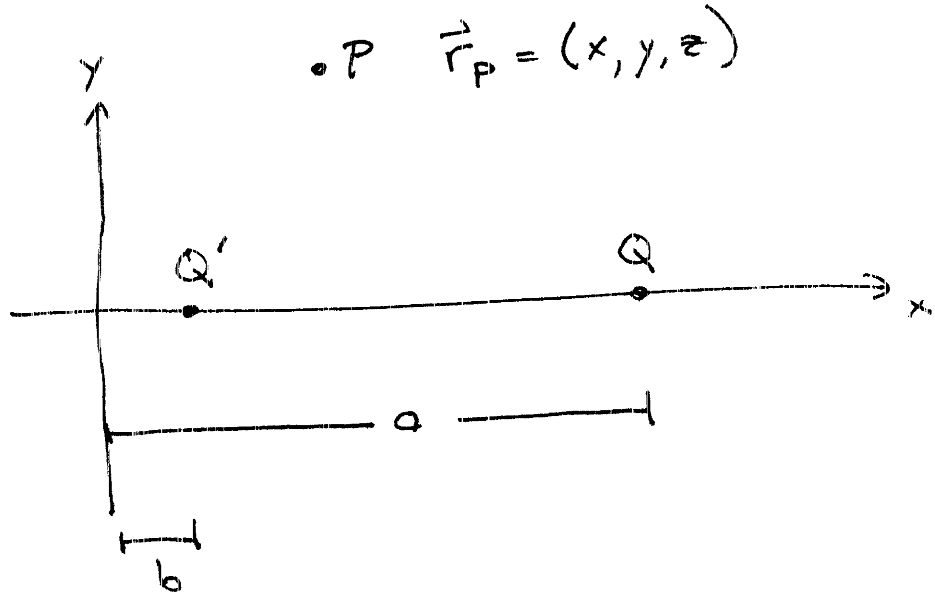


Image Charge $Q' = -\frac{R}{a} Q$

at $b = \frac{R^2}{a}$

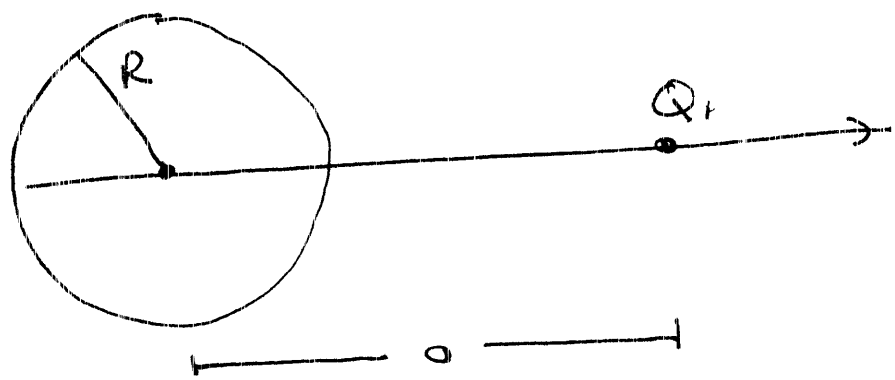
Evidently, the sphere has a non-zero total charge. Using Gauss' Law, the total charge of the sphere must be Q'.

The potential outside the sphere



$$V(x,y,z) = \frac{kQ}{\sqrt{(x-a)^2+y^2+z^2}} + \frac{kQ'}{\sqrt{(x-b)^2+y^2+z^2}}$$

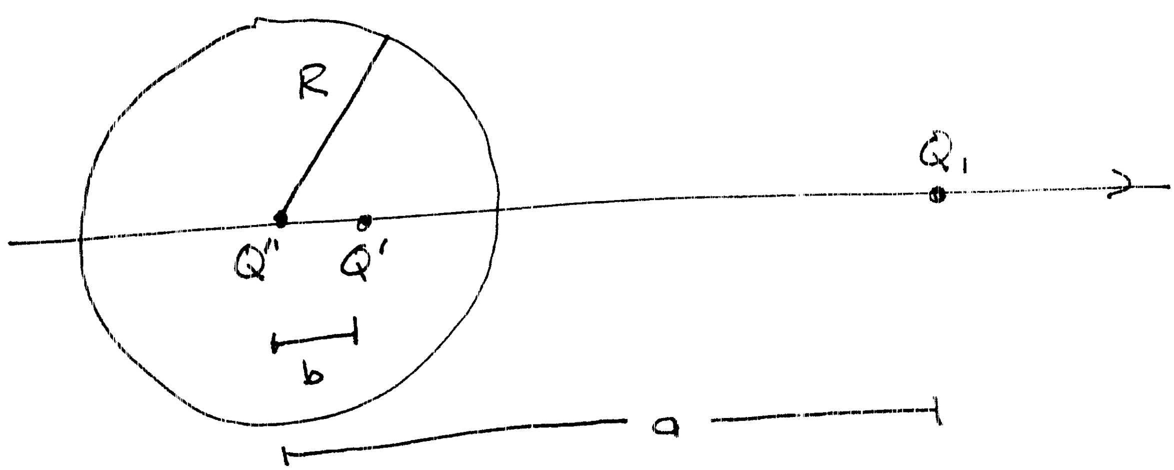
<u>Ex</u> Charged Conducting Sphere, total charge Q, and point charge with charge Q_1



Once again use an image charge $Q' = -\frac{R}{a} Q_1$
at location $b = \frac{R^2}{a}$ to bring the surface of
the conductor to zero potential. This leaves the
conductor with net charge Q'. Spread a uniform
surface charge with total charge

$$Q'' = Q - Q'$$

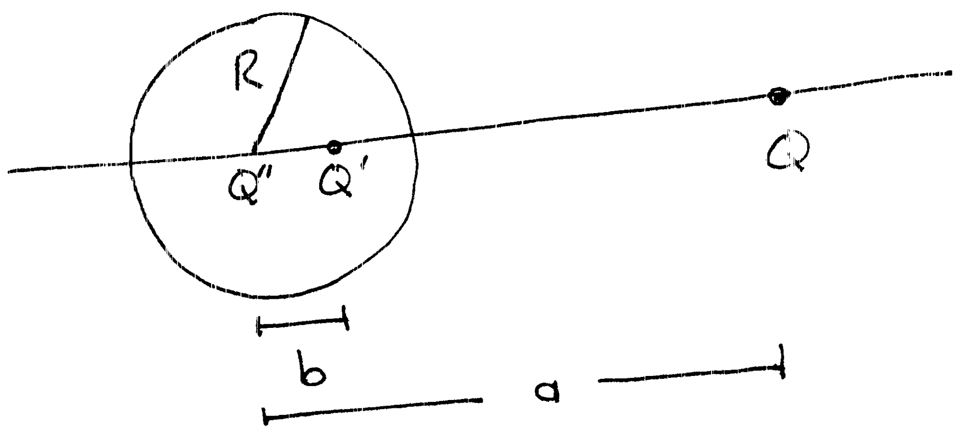
over the surface of the sphere. A uniform surface
charge will work because the surface is all at
the same potential. This produces the same potential
as a charge Q'' at the center of the system. Our
finished system is



Ex

This allows us to answer a question that has always plagued many of you. What is the actual force exerted by one pith ball on another pith ball?

A charged conducting sphere with charge Q exerts a force F on a point charge Q. Same system as above except $Q_1 = Q$.



$$F = \frac{kQQ'}{(a-b)^2} + \frac{kQQ''}{a^2}$$ (positive if force points to right)

$$b = \frac{R^2}{a} \qquad Q' = -\frac{R}{a} Q$$

$$Q'' = Q - Q' = Q\left(1 + \frac{R}{a}\right)$$

$$F = -KQ^2\left(\frac{R}{a(a-b)^2}\right) + KQ^2\left(\frac{1}{a^2} + \frac{R}{a^3}\right)$$

$$= KQ^2\left(\frac{-Ra^2 + a(a-b)^2 + (a-b)^2R}{a^3(a-b)^2}\right)$$

$$= \frac{KQ^2}{a^2}\left(\frac{a(a-b)^2 + b^2R - 2abR}{a(a-b)^2}\right)$$

But $b = R^2/a$

$$F = \frac{KQ^2}{a^2}\left(\frac{a(a-b)^2 + R^5/a^2 - 2R^3}{a(a-b)^2}\right)$$

$$(a-b)^2 = \left(a - \frac{R^2}{a}\right)^2 = a^2 - 2R^2 + \frac{R^4}{a^2}$$

$$F = \frac{KQ^2}{a^2}\left(\frac{a^3 - 2aR^2 + R^4/a + R^5/a^2 - 2R^3}{a^3 - 2aR^2 + R^4/a}\right)$$

Sometime it doesn't get pretty.

If both spheres are approximated as point charges the force is kQ^2/a^2. So the correction is

$$\frac{F_{sphere}}{F_{point}} = \frac{a(a-b)^2 + b^2R - 2abR}{a(a-b)^2}$$

For our pith balls, $R \sim 1/4$ cm $\quad a \sim 2$ cm

$$b = R^2/a = \frac{\frac{1}{16}}{2} \text{ cm} = \frac{1}{32} \text{ cm}$$

$$\frac{F_{sphere}}{F_{point}} = \frac{(2)\left(\frac{63}{32}\right)^2 + \left(\frac{1}{32}\right)^2 \cdot \frac{1}{4} - 2(2)\left(\frac{1}{32}\right)\left(\frac{1}{4}\right)}{2\left(\frac{63}{32}\right)^2}$$

$$= \frac{2(63)^2 + 1/4 - 32}{2(63)^2} = 0.996$$

So the point charge approximation is very good.

We can also use a point charge in the middle of the sphere to raise the potential at the surface to V_0.

Image Charge for Conducting Cylinder

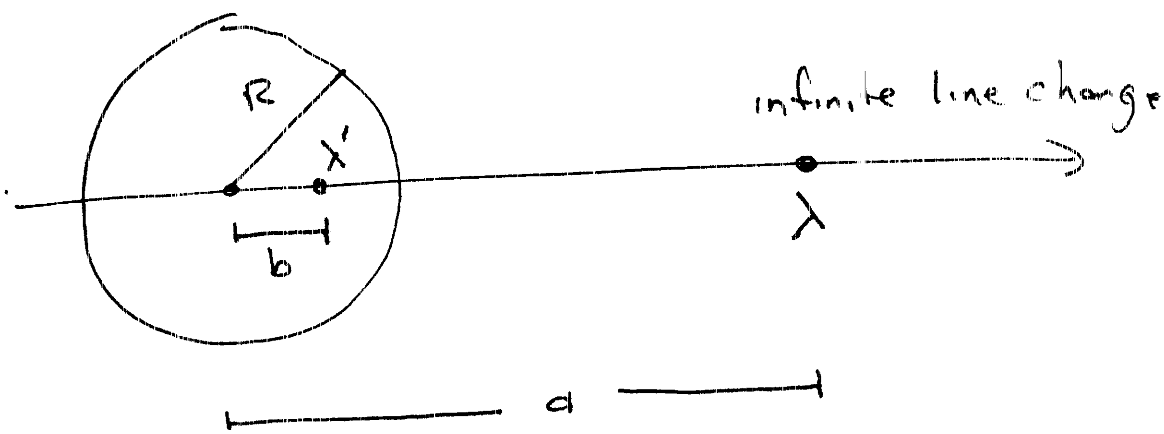


Image Line Charge $\lambda' = \lambda$ at

$$b = \frac{R^2}{a}$$