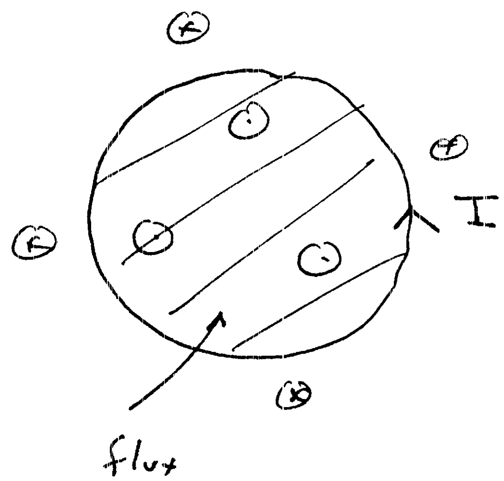
The magnetic field is proportional to the current, therefore the flux through some surface S is proportional to the current that produces the field.

<u>Self-Inductance</u> (L) - The ratio of the flux through a circuit to the current in the same circuit that produced the flux.



$$L = \frac{\Phi_m}{I}$$

Mutual Inductance (M_{ab}) - The ratio of the flux through circuit b to the current in circuit a that produced the flux.

$$M_{ab} = \frac{\Phi_b}{I_a} = M_{ba}$$

$\Rightarrow$ Both L and M_{ab} are independent of current and depend only on ~~ge~~ constants and geometry.

$\Rightarrow$ Units Henry

$$1\,H = \frac{1\,T m^2}{A}$$

Let's compute M_{ab} in general

$$\Phi_b = \oint_{C_b} \vec{A}_a \cdot d\vec{l}_b$$

<u>Vecter Potential</u>

$$\vec{A}_a = \frac{\mu_0 I_a}{4\pi} \oint_{C_a} \frac{d\vec{l}_a}{r''}$$

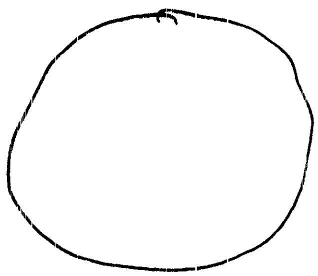
<u>therefore</u>

$$\Phi_b = \frac{\mu_0 I_a}{4\pi} \oint_{C_b} \oint_{C_a} \frac{d\vec{l}_a \cdot d\vec{l}_b}{r''}$$

$$M_{ab} = \frac{\Phi_b}{I_a} = \underbrace{\frac{\mu_0}{4\pi} \oint_{C_b} \oint_{C_a} \frac{d\vec{l}_a \cdot d\vec{l}_b}{r''}}_{\text{Geometry + Constants.}}$$

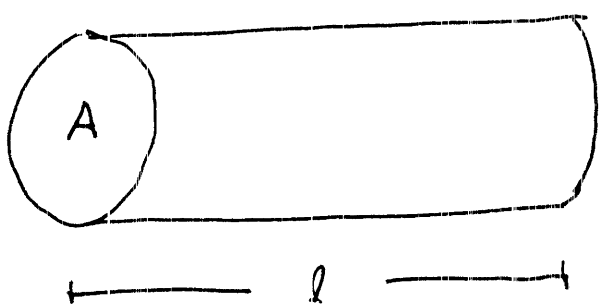
$$= M_{ba}$$

Self-Inductance - Ring



- Too hard

Solenoid - Assume fields are the same as that of infinite solenoid.



N turns

Let current I flow in solenoid.

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 n I$$

$$n \equiv \frac{N}{\ell}$$

Flux

$$\Phi_m = N B A$$

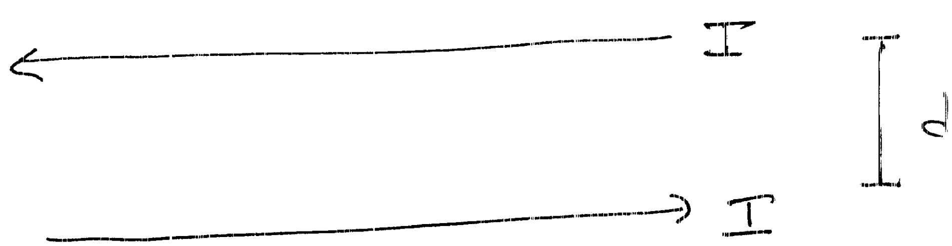
$$= \mu_0 N n I A$$

$$= \mu_0 n^2 I A l$$

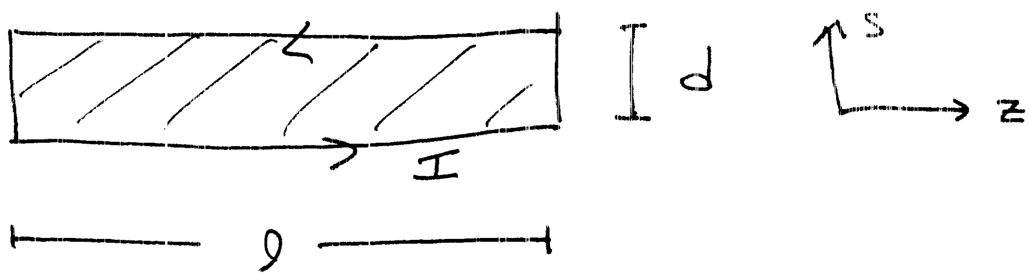
Inductance

$$L = \frac{\Phi_m}{I} = \mu_0 n^2 A l$$

Infinite Parallel Wires



Close the path at length l, $l >> d$.



Let wire have radius a.

Let current I flow in the circuit.

The total field between the wires is

$$B = \frac{\mu_0 I}{2\pi s} + \frac{\mu_0 I}{2\pi (d-s)}$$

The total flux through the area between the loop is

$$\Phi_m = l \int_a^{d-a} B \, ds$$

$$\cancel{= \frac{\mu_0 I l}{2\pi} \left[\ln\left(\frac{d-a}{d}\right) - \ln\left(\right. \right.}$$

Each wire makes same contribution to the flux.

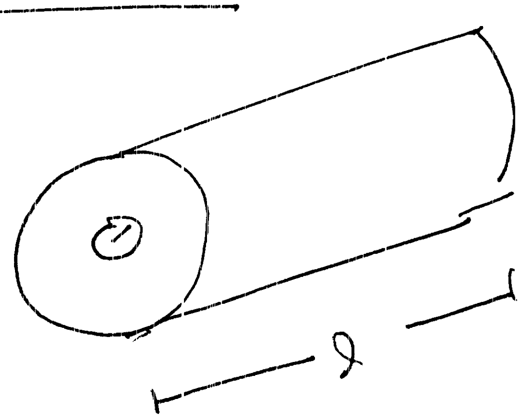
$$\Phi_m = 2l \int B\, ds = 2l \frac{\mu_0 I}{2\pi} \int_a^{d-a} ds/s$$

$$= \frac{l \mu_0 I}{\pi} \ln\left(\frac{d-a}{a}\right)$$

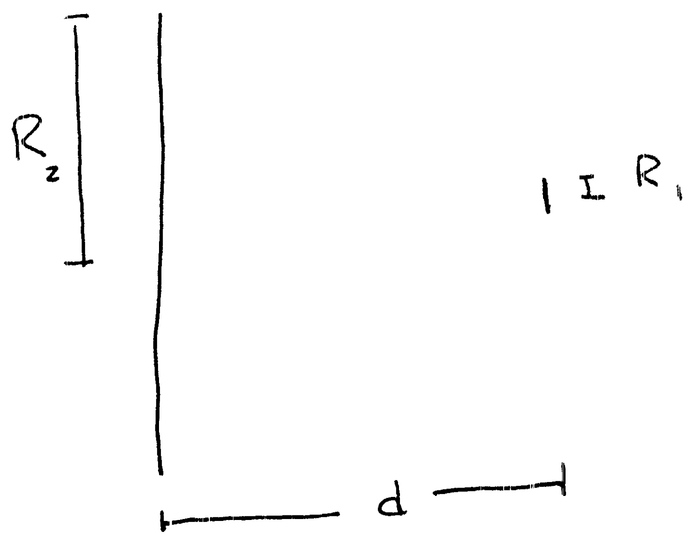
Inductance

$$L = \frac{\Phi_m}{I} = \frac{\mu_0 l}{\pi} \ln\left(\frac{d-a}{a}\right)$$

Co-axial Cable



<u>Mutual Inductance</u> - Compute mutual inductance of two rings $R_1 \ll R_2$ a distance d apart. The rings are co-axial.



We can either compute M_{21} or M_{12}, one is much easier.

<u>Compute M_{12}</u> -

$$M_{12} = \frac{\Phi_1}{I_2} = M_{21}$$

Since R_1 is small, the field of R_2 is approximately constant at R_1

$$\Phi_1 \approx \pi R_1^2 B_2$$

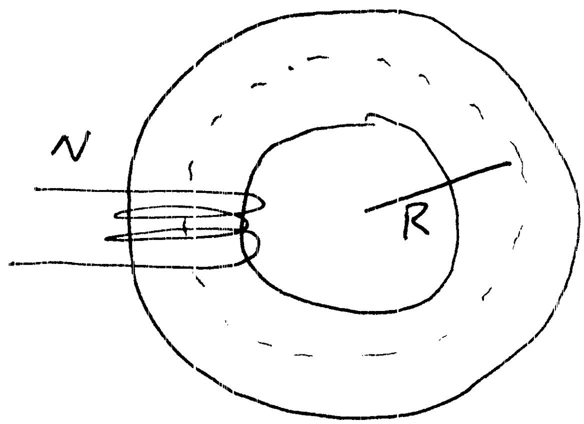
$$B_2 = \frac{\mu_0 I_2 R_2^2}{2(d^2 + R_2)^{3/2}}$$

$$M_{12} = \frac{\Phi_1}{I_2} = \frac{\mu_0 \pi R_1^2 R_2^2}{2(d^2 + R_1^2)^{3/2}}$$

If we calculated M_{21} we would have to integrate the field over the loop.

<u>Ex</u> Self-Inductance of Iron Ring,

N turns of wire, cross section A.



<u>Compute Field</u>

$$\oint \vec{H} \cdot d\vec{l} = NI = 2\pi R H$$

$$B = \mu_0 \mu_r H$$

$$B = \frac{\mu_0 \mu_r NI}{2\pi R}$$

Flux

$$\Phi_m = NBA$$

$$= NA\left(\frac{\mu_0 \mu_r N I}{2\pi R}\right)$$

$$= \frac{\mu_0 \mu_r N^2 A I}{2\pi R}$$

Self-Inductance

$$L = \frac{\Phi_m}{I} = \frac{\mu_0 \mu_r N^2 A}{2\pi R}$$