Linear Magnetic Materials

Some materials like copper and aluminum have a linear response to an applied field. Unfortunately, iron does not have a linear response.

$$\vec{M} = \chi_m \vec{H}$$

<u>Magnetic Susceptibility</u> (χ_m) - Characterizes a materials linear response to applied field.

$$\mu_r \equiv 1 + \chi_m \quad \text{Relative Permeability}$$

$$\mu = \mu_r \mu_0 \quad \text{Permeability}$$

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0 \vec{H} + \mu_0 \chi_m \vec{H}$$

$$= \mu_0 (1 + \chi_m) \vec{H}$$

$$= \mu_0 \mu_r \vec{H}$$

$$= \mu \vec{H}$$

Paramagnetic ($\chi_m > 0$) $\Rightarrow$ Magnetic moment in same direction as field, $\mu_r > 1$.

Diamagnetic ($\chi_m < 0$) $\Rightarrow$ Magnetic moment opposite direction as field, $\mu_r < 1$.

Example

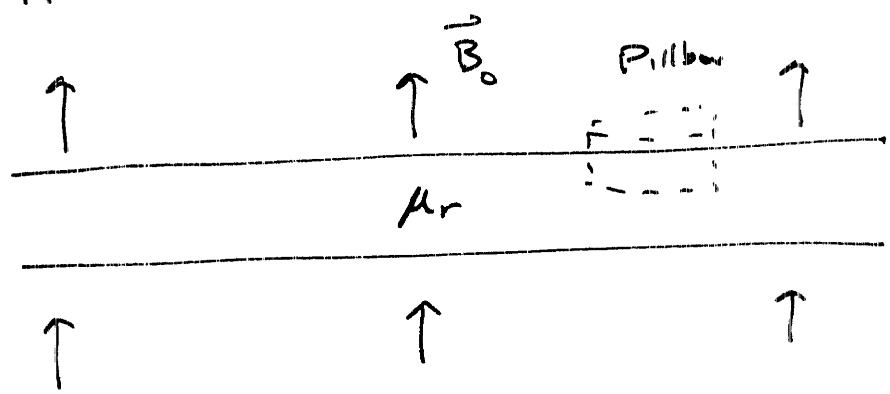
χ_m	copper	-1×10^{-5}
χ_m	polyethylene	-0.2×10^{-5}
χ_m	aluminum	2.1×10^{-5}

Note, other materials are extremely non-linear and have large χ_m

iron $\chi_m \sim 1000$

superconductor $\chi_m \equiv -1$

Ex Thin magnetic slab placed in uniform applied field $\vec{B}_0$



Outside $\vec{M} = 0$ $\mu_0 \vec{H}_0 = \vec{B}_0$

Inside Using Pillbox

$$\nabla \cdot \vec{B} = 0 \Rightarrow \Phi_m = 0$$

$$\Rightarrow \vec{B}_0 = \vec{B}_i$$

$$\vec{B}_i = \mu_0 \vec{H}_i + \mu_0 \vec{M}$$

$$= \mu \vec{H}_i \qquad \text{because linear}$$

$$= \vec{B}_0$$

$$\vec{H}_i = \frac{\vec{B}_0}{\mu} = \frac{\vec{H}_0}{\mu_r}$$

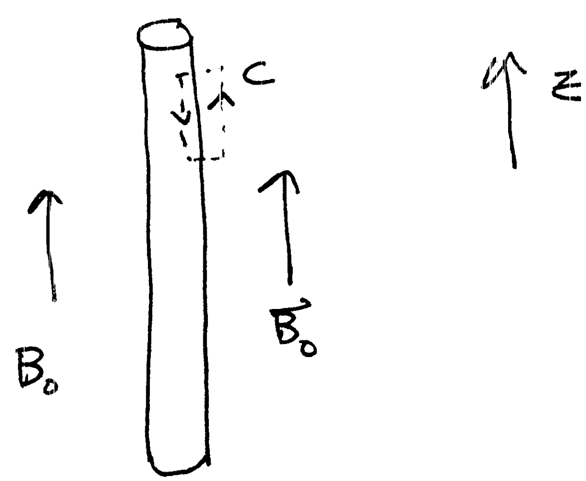
The slab reduces ($\chi_m > 1$) the applied field by a factor of μ_r.

Magnetization

$$\vec{M}_i = \chi_m \vec{H}_i = \frac{\chi_m}{\mu_r} \frac{\vec{B}_0}{\mu_0}$$

$$= \frac{\chi_m}{1+\chi_m} \frac{\vec{B}_0}{\mu_0}$$

Ex Place needle in uniform applied field



Pillbox $B_0^{\perp} = B_i^{\perp} = 0$ ($\vec{B}$ must be in $\hat{z}$ direction)

Amperian Path

$$\oint \vec{H} \cdot d\vec{l} = 0 \quad (I_{f\,enc} = 0)$$

$$H_i^{\parallel} = H_0^{\parallel}$$

$$\vec{H}_o = \frac{\vec{B}_o}{\mu_o} = \vec{H}_i$$

$$\vec{M}_i = \chi_m \vec{H}_i = \chi_m \frac{\vec{B}_o}{\mu_o}$$

$$\vec{B}_i = \mu \vec{H}_i = \mu_o \mu_r \frac{\vec{B}_o}{\mu_o} = \mu_r \vec{B}_o$$

Field inside is increased ($\chi_m > 1$) by μ_r.

<u>Ex</u> Suppose cylinder is made of iron that has relative permeability $\mu_r = 1000$ at the field produced by placing the nail in the Earth's field $B_e = 4 \times 10^{-5}\,T$

$$B_{nail} = \mu_r B_{earth} = 4 \times 10^{-2}\,T$$

$$M_{nail} = \chi_m \frac{\vec{B}_o}{\mu_o} = (1000) \frac{4 \times 10^{-5}\,T}{4\pi \times 10^{-7}\,\frac{Tm}{A}}$$

$$= 32{,}000\ A/m$$

If nail is 10cm long by 1mm in radius, the total dipole moment is

$$m = MV = M \pi r^2 h = (32{,}000\,A/m)(0.1\,m)\pi(0.001\,m)^2$$
$$= 0.003\,A\,m^2$$

Surface Current

$$|\vec{K}| = |\vec{M} \times \hat{n}| \quad 32,000 \text{ A/m}$$

Total surface current

$$|\vec{K}| h = 3,200 \text{ A}$$

Why do slab and needle have such different responses? The effects are caused by surface currents, the slab minimizes the current, the needle maximizes the current.