Magnetic Materials

Magnetization (Density) ($\vec{M}$) - Magnetic dipole moment per unit volume.

Dipole Vector Potential

$$\vec{A}_{dip} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

Vector Potential of Object with Magnetization $\vec{M}$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}\,') \times \hat{r}''}{r''^2} \, d\tau'$$

Use the same process we used on polarization

$$\nabla' \left(\frac{1}{r''} \right) = \frac{\hat{r}''}{r''^2}$$

so

$$\vec{A} = \frac{\mu_0}{4\pi} \int \vec{M} \times \nabla' \left(\frac{1}{r''} \right) d\tau'$$

Vector Identity

$$\nabla \times (f\vec{A}) = f(\nabla \times \vec{A}) - \vec{A} \times \nabla f$$

Let $f = \frac{1}{r''}$, $\vec{A} = \vec{M}$

$$\nabla' \times \left(\frac{\vec{M}}{r''}\right) = \frac{1}{r''} \nabla' \times \vec{M} - \vec{M} \times \nabla'\left(\frac{1}{r''}\right)$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \vec{M} \times \nabla'\left(\frac{1}{r''}\right) d\tau'$$

$$= \frac{\mu_0}{4\pi} \int \frac{\nabla' \times \vec{M}}{r''} d\tau' - \frac{\mu_0}{4\pi} \int \nabla' \times \left(\frac{\vec{M}}{r''}\right) d\tau'$$

Curl Divergence Thm

$$\int_V \nabla \times \vec{A} \, d\tau = - \oint_S \vec{A} \times \hat{n} \, da$$

So

$$\vec{A} = \frac{\mu_0}{4\pi} \int_V \frac{\nabla' \times \vec{M}(\vec{r}')}{r''} d\tau' + \frac{\mu_0}{4\pi} \oint_S \frac{\vec{M}(\vec{r}') \times \hat{n}'}{r''} da'$$

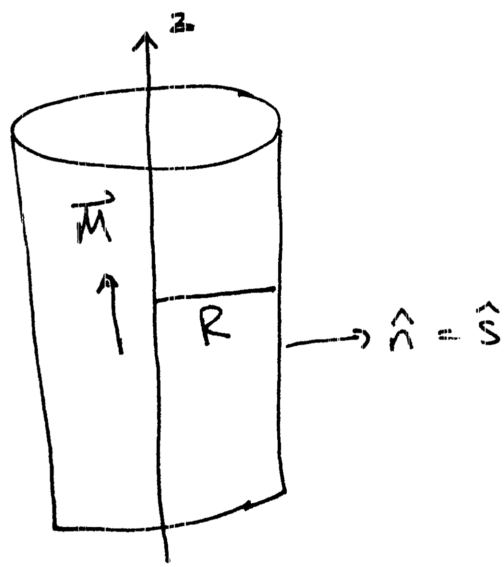
Define Bound Currents

$$\vec{J}_b = \nabla \times \vec{M} \qquad \text{volume}$$

$$\vec{K}_b = \vec{M} \times \hat{n} \qquad \text{surface}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}_b}{r''} \, d\tau' + \frac{\mu_0}{4\pi} \int \frac{\vec{K}_b}{r''} \, da'$$

Ex Infinite Cylinder with Magnetization $\vec{M} = M_0 \hat{z}$



Compute Fields, Currents

$$\vec{J}_b = \nabla \times \vec{M} = 0$$

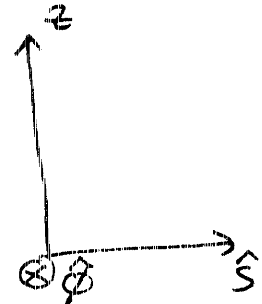
$$\vec{K}_b = M \times \hat{n}$$

Since infinitely long, we only have to worry about sides.

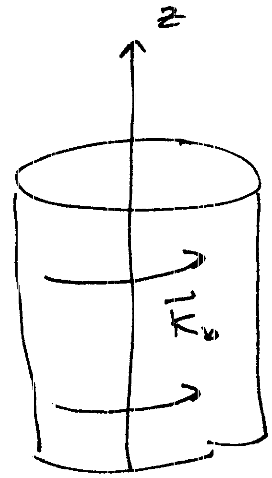
$\vec{M} = M_0 \hat{z}$ $\quad\quad \hat{n} = \hat{s}$

$\vec{K}_b = \vec{M} \times \hat{n} = (M_0 \hat{z}) \times \hat{s}$

$= M_0 \hat{\phi}$



$\hat{z} = \hat{s} \times \hat{\phi}$



Once we have the bound currents, we can ignore the magnetic material.

The current density found is just that of an infinite solenoid, so the fields are:

Outside: $\vec{B}_o = 0$

Inside: $\vec{B}_i = \mu_0 K_b \hat{z} = \mu_0 M_0 \hat{z}$

<u>How much field / current?</u> For NdFeB magnets,

$$M_0 = 1.02\times10^6 \,A/m$$

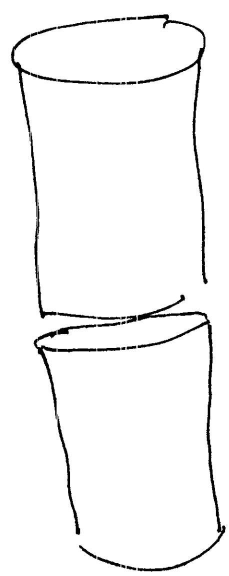
$$K_b = M_0$$

So if the magnet was one meter long, 10^6 Amps of bound current flow producing a field

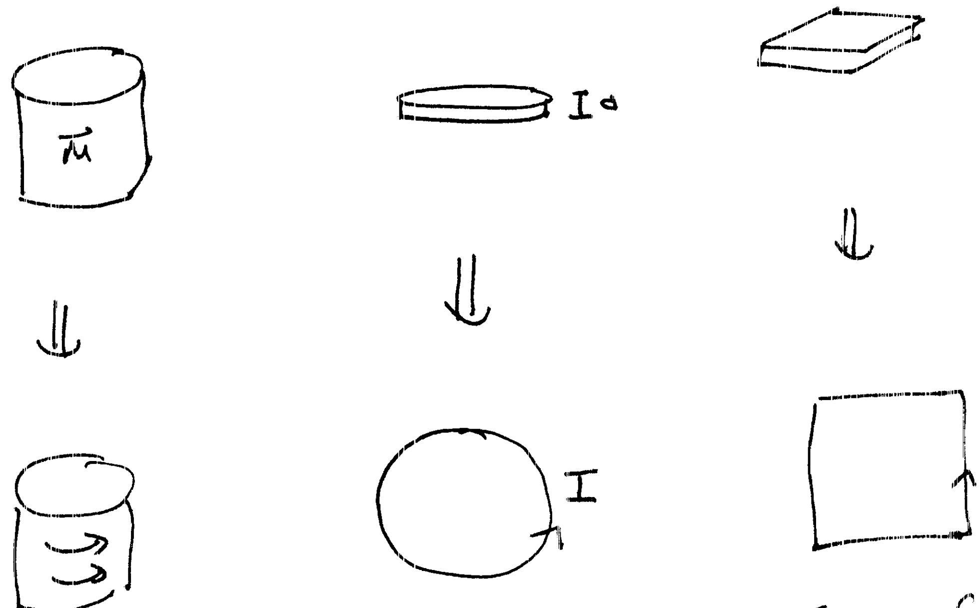
$$B_i = \mu_0 K_b = \left(4\pi\times10^{-7}\frac{Tm}{A}\right)\left(1.02\times10^6 A/m\right)$$

$$= \cancel{0.128\,T} \quad 1.28\,T$$

Since there is no magnetic charge, this is the field in a small gap in a long magnet



Infinitely long magnets are fairly uncommon, but with tools we already have we can calculate the field of a variety of common magnet geometries.



Finite Solenoid

Ring of current

$I = k_b a$

Square Current Loop.

Note, if the magnetization is uniform, then the total magnetic moment is $\vec{m} = \vec{M} V$ where V is the volume. We can then use the dipole fields far from the magnet.