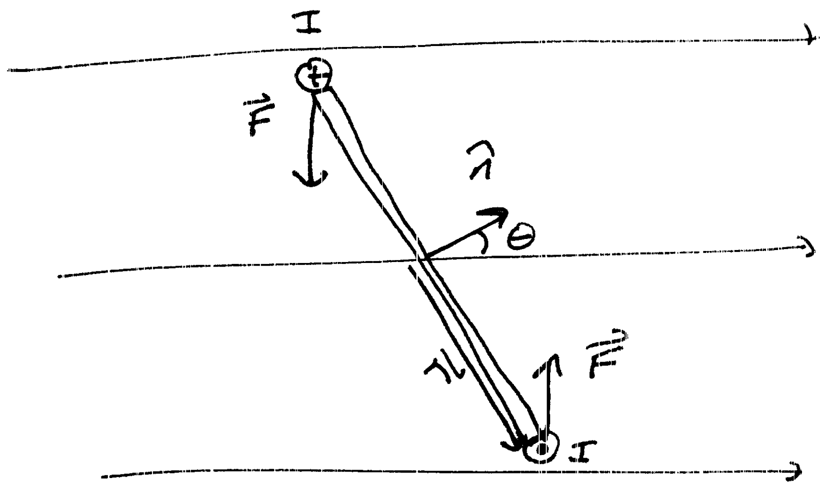
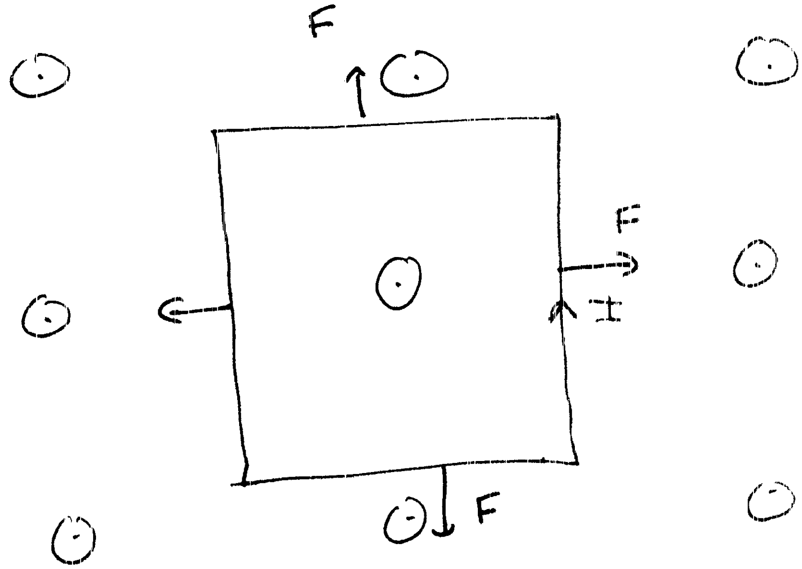
Force and Torque on Magnetic Dipole

Consider a square loop of wire whose normal makes an angle θ with a uniform magnetic field $\vec{B} = B\hat{y}$.





There is no net force on the loop, but there is a net torque as drawn above.

If the loops is an $l \times l$ squre,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = \frac{l}{2} \sin\theta \, F \quad \text{of one wire}$$

<u>Total Torque</u> $= 2|\vec{\tau}| = l \sin\theta \, F$

$$F = I l B$$

$$\tau_{total} = \underbrace{I l^2}_{m} B \sin\theta$$

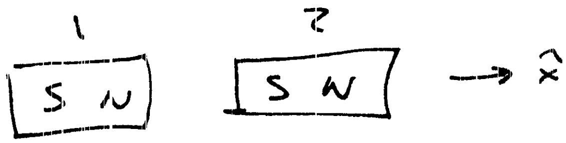
$$\boxed{\vec{\tau} = \vec{m} \times \vec{B}}$$ Torque on point dipole

Integration the torque, gives the potential energy of point dipole.

$$U = -\vec{m} \cdot \vec{B}$$

If the field is non-uniform, the total force is $\vec{F} = \nabla(\vec{m} \cdot \vec{B})$

This explains why magnets snap together.



The field of magnet 1 has dependence $B = \frac{\alpha}{r^3}$ and has little effect on the moment of magnet 2.

$$\vec{F} = \nabla(\vec{m}\cdot\vec{B}) = \frac{\partial}{\partial x}\left(\frac{m_o \alpha}{r^3}\right)\hat{x}$$

$$= -3\frac{m_o \alpha}{r^4}\hat{x} \quad \text{(not attractive)}$$

This means the force increases as r^4 as the magnets get closer. Naturally, you have to correct for the finite size of the magnets but r^4 is a very strong dependence.