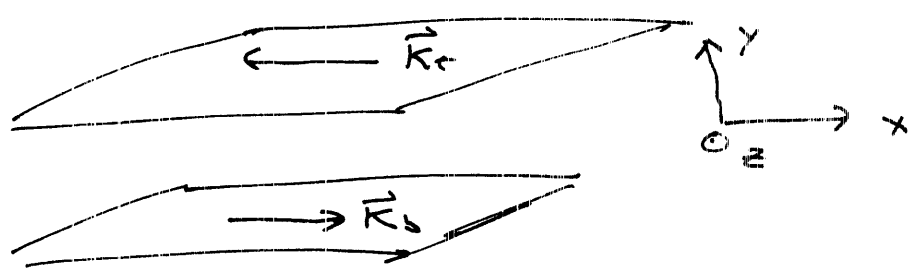
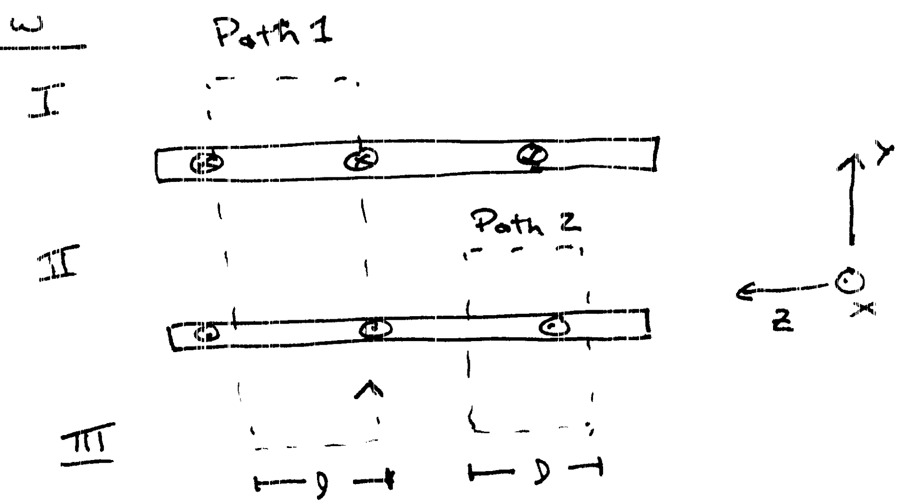
A bit more Vector Potential

Consider a system with two current sheets



$\vec{K}_t = -K_0\hat{x}$ $\quad\vec{K}_b = K_0\hat{x}$

End View



Path 1 Out of the page $+\hat{n} = \hat{x}$

$$I_{enc} = -K_0\ell + K_0\ell = \vec{K}_t\cdot\hat{n}\ell + \vec{K}_b\cdot\hat{n}\ell$$

$$= 0$$

By the fact the fields do not decay with distance the field must be equal, but opposite.

$$\vec{B}_{III} = -\vec{B}_{I}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} = 0 = -B_{III}\, l + B_{I}\, l$$

$$B_{III} = B_{I}$$

$$\Rightarrow \vec{B}_{I} = \vec{B}_{III} = 0$$

<u>Path II</u> $\quad I_{enc} = \vec{K}_s \cdot \hat{n}\, l = K_0\, l$

$$\cancel{\oint \vec{B} \cdot d\vec{l} = B_I\, l - B_{II}\, l =}$$

Let's be a little more careful, define

$$\vec{B}_{I} = B_{I}\,\hat{z} \qquad \vec{B}_{II} = B_{II}\,\hat{z}$$

note from diagram, $\hat{z}$ points to left.

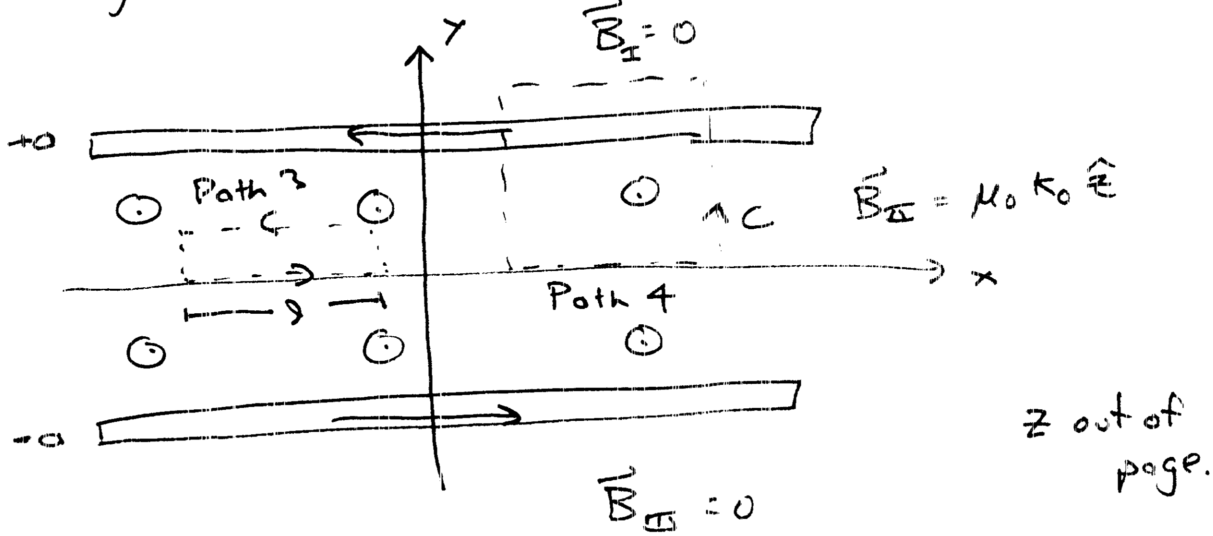
$$\oint \vec{B} \cdot d\vec{l} = \underbrace{-B_{I}\, l}_{=\,0} + B_{II}\, l = \mu_0 I_{enc} = \mu_0 K_0\, l$$

$$\vec{B}_{II} = \mu_0 K_0\, \hat{z}$$

Compute Vector Potential   Let origin be point halfway between planes. Choose (and I can because its a potential) $\vec{A}(0,0,z) = 0$.

$$\oint \vec{A}\cdot d\vec{l} = \Phi_m$$

Redraw diagram



Outward normal out of the page by RHR.

<u>Path 3</u>
$$\Phi_m = \int \vec{B}\cdot\hat{n}\, dA = B_{II}\, y\, l$$

$$= \oint_C \vec{A}\cdot d\vec{l} = \vec{A}(\cancel{x},0,z)\cdot\hat{x}\, l - \vec{A}(\cancel{x},y,z)\cdot\hat{x}\, l$$

$$= -A(\cancel{x},y,z)\, l \equiv -A_{II}\, l$$

<u>Inside</u> $\vec{A}_{II} = -B_{II}\, y\,\hat{x} = -\mu_0 K_0\, y\,\hat{x}$

$\vec{A}$ is continuous, so

$$\vec{A}_{III} = -\mu_0 k_0 a \, \hat{x}$$

$$\vec{A}_{I} = \mu_0 k_0 a \, \hat{x}$$

Check the field is recovered

$$\nabla \times \vec{A} = \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -B_{II} y & 0 & 0 \end{vmatrix}$$

$$= B_{II} \, \hat{z} \quad \checkmark$$

For connection with other classes. As we let the electromagnetic field change, we will use the definition

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \nabla \times \vec{A}$$

For this situation, the classical Hamiltonian, the total energy written in terms of position and momentum ($\vec{p}$) is

$$H = \frac{1}{2m} \left(\vec{p} - q\vec{A} \right)^2 + qV$$

In quantum mechanics, Schrodinger eqn is

$$i\hbar \frac{\partial \psi}{\partial t} = H \psi$$

$$H = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\vec{A}\right)^2 + qV$$

For example, if you wished to compute the quantized energy of an electron moving in a circular orbit in a solenoid.

$$\vec{A} = \gamma_0 s \hat{\phi} \qquad V = 0$$