Overview

From introductory physics

Maxwell's Eqns

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$ Gauss'

$$\oint \vec{B} \cdot d\vec{a} = 0$$ No Magnetic Monopoles

$$\oint_c \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_s \vec{E} \cdot d\vec{a}$$ Ampere

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_s \vec{B} \cdot d\vec{a}$$ Faraday

Coulomb's Law

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2} \hat{r}$$

Biot-Savart Law

$$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

Lorentz Force

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

Wave Solution

$\exists$ wave solution to Maxwell's eqns that travels at speed of light.

What are we going to do?

(1) Use Cal III to turn Maxwell's eqns into differential eqns.

(2) Investigate new solution techniques the differential equations allow.

(3) Introduce more natural methods to handle dielectrics, conductors, and magnetic materials.

(4) Investigate conservation laws.

(5) Investigate wave solution in the presence of materials.

Vectors

Scalar - An object with magnitude but not direction. Example temperature

Vector - An object with magnitude and direction.
Example - Heat flow

- Transforms the same way as the position vector $\vec{r} = (x, y, z)$ under 3D rotations.

- Representing vectors $\vec{A}$

$$\vec{A} = (A_x, A_y, A_z)$$

$$= A_x \hat{x} + A_y \hat{y} + A_z \hat{z} \quad \text{(preferred)}$$

$$= A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}$$

- Unit Vectors - Length 1

$$\hat{x} = (1, 0, 0)$$

Not all three-tuples are vectors, but to see this we need to be able to rotate stuff.

<u>Matrices</u> ($\overleftrightarrow{T}$) - Vectors are rotated, or subjected to other changes by matrices.

$$\overleftrightarrow{T} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$$

For example, to rotate a vector by an angle θ about the $\hat{z}$ axis

$$\overleftrightarrow{T}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ +\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If an object is a vector, then rotating the object in real life produces the same effect as applying the rotation matrix.

Consider the position vector,

$$\vec{A} = (A_x, A_y, A_z) = (1m, 2m, 0)$$

If we rotate the vector by $45°$ in real life we get the vector $\vec{A}\,' = (0, \sqrt{2}m, 0)$.

If $\vec{A}$ is a vector, it will mathematically be rotated by

$$\overleftrightarrow{T}_{45°} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\vec{A}\,' = \overleftrightarrow{T}_{45°} \vec{A}$$

$$\begin{pmatrix} A_x' \\ A_y' \\ A_z' \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1m \\ 1m \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \sqrt{2}\, m \\ 0 \end{pmatrix} \checkmark$$

Not all 3-tuples are vectors, if we did the same thing in a zoo with

$$\vec{A} = (2 \text{ lions}, 3 \text{ tiger}, 2 \text{ bears})$$

we would not get

$$\vec{A} = \left(\frac{2 \text{ lions} - 3 \text{ tigers}}{\sqrt{2}}, \frac{2 \text{ lions} + 3 \text{ tigers}}{\sqrt{2}}, 2 \text{ bears} \right)$$

<u>Symmetry</u> - A symmetry of a system is a transformation that leaves the system unchanged.

<u>Example</u> - A linear object lying along the z-axis with uniform mass density has cylindrical symmetry, it is unchanged under rotations $\overleftrightarrow{T}_\theta$ about z-axis.

<u>Example</u> - A point object has spherical symmetry. It is unchanged by rotations about <u>ANY</u> axis.

There are other transformations we can apply to an object.

<u>Reflection</u> Reflect an object through x-y plane

$$\vec{A}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

<u>Inversion</u> - Reflection through origin

$$\vec{A}' = -\vec{A}$$

$$= \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

<u>Psvedo Vectors</u> All vectors must behave well under rotation, or they aren't vectors. Vectors that don't change sign under inversion are called psvedo vectors. Many vectors defined by a cross product are psvedo vectors.

If $\vec{A} \to -\vec{A}$ and ~~B_y~~ $\vec{B} \to -\vec{B}$ under inversion

$$\vec{C} = \vec{A} \times \vec{B} \to \vec{C}$$

Vector Products

Dot product

$$\vec{A} \cdot \vec{B} = \text{scalar}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$= |\vec{A}||\vec{B}| \cos\theta$$

Vector Length (Modulus)

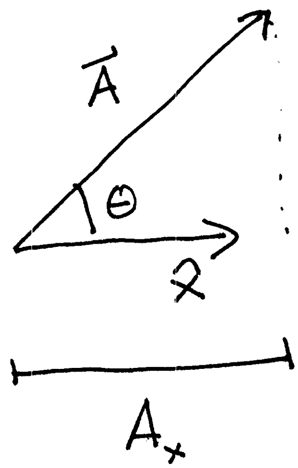
$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \equiv A$$

$$|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}} \equiv A$$

Unit Vector - Vector of unit length $\hat{A}$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

Dot products with unit vector project along the direction of the unit vector. Suppose we want the component of $\vec{A}$ along the x-direction



$$A_x = \vec{A} \cdot \hat{x} = |\vec{A}| \cos\theta = |\vec{A}| \underbrace{|\hat{x}|}_{=1} \cos\theta$$

<u>Cross - products</u> — The cross-product of $\vec{A}$ and $\vec{B}$ returns a <u>vector</u> $\perp$ to both $\vec{A}$ and $\vec{B}$.

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

$$\vec{C} = \vec{A} \times \vec{B} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- Cross-product does not commute

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

- $\vec{A} \times \vec{A} = 0$

- Direction - Right-hand Rule - $\vec{C} = \vec{A} \times \vec{B}$

  thumb

  curl fingers

  Point fingers toward $\vec{A}$ curl fingers ($<180°$) toward $\vec{B}$, thumb points toward $\vec{C}$.

- Cross-product does not ~~distribute~~ associate ! You are asked to prove.

$$\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0$$

so

$$\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) = -\vec{C} \times (\vec{A} \times \vec{B})$$

$$= (\vec{A} \times \vec{B}) \times \vec{C}$$

so be careful

- $\vec{A} \times \vec{B} \times \vec{C}$ has no meaning, it has to have ( ).

Triple Products

Scalar $\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$

$$= \det\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

- Volume of parallel-piped formed by $\vec{A}, \vec{B}, \vec{C}$

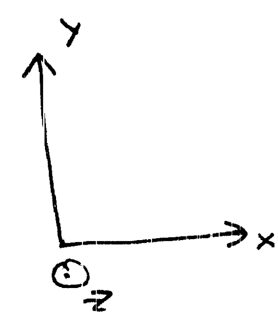
Vector $\vec{A}\times(\vec{B}\times\vec{C})$

BAC - CAB Rule

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

Proof Cross-product not associative

$$(\hat{y}\times\hat{x})\times\hat{x} = (-\hat{z})\times\hat{x}$$
$$= -\hat{y}$$



$$\hat{y}\times(\hat{x}\times\hat{x}) = 0$$