Energy and Momentum

EM waves are just electric and magnetic fields and we know how to compute the energy and momentum in these fields.

Energy Density

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

$$B = E/c$$

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0 c^2} E^2$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{\mu_0 \varepsilon_0}{2\mu_0} E^2$$

$$= \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\varepsilon_0 E^2 = \varepsilon_0 E^2$$

The electric field oscillates in time,

$$u = \varepsilon_0 E^2 = E_0^2 \varepsilon_0 \cos^2(kx - \omega t + \sigma)$$

Time Averaged Energy Density

$$\langle u \rangle = \frac{1}{T} \int_0^T u \, dt$$

where T is one period of oscillation.

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\langle u \rangle = \varepsilon_0 \frac{1}{T} E_0^2 \int_0^{\frac{2\pi}{\omega}} \cos^2(kx - \omega t + \sigma) \, dt$$

$$= \frac{1}{2} \varepsilon_0 E_0^2$$

Note - $kx + \sigma$ acts as a phase in the integral and does not matter.

Average Energy Density in Electromagnetic Wave

$$\langle u \rangle = \frac{1}{2} \varepsilon_0 E_0^2$$

Intensity (I) - Energy per unit area per unit time

$\Longrightarrow$ Energy flux.

The energy flux in a EM field is given by the Poynting vector, $I = |\vec{S}|$

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$$

$$= \frac{1}{\mu_0}\left(E_0\right)\left(\frac{E_0}{c}\right)\cos^2(kx - \omega t + \delta)\,\hat{z}$$

energy flow must be in propagation direction

$$= \varepsilon_0 c E_0^2 \cos^2(\;)\,\hat{z}$$

$$\boxed{\begin{aligned} \frac{1}{\mu_0 \varepsilon_0} &= c^2 \\ \frac{1}{\mu_0} &= \varepsilon_0 c^2 \end{aligned}}$$

The time average of $\vec{S}$ gives an $1/2$ as before

$$\langle I \rangle = \langle |\vec{S}| \rangle = \frac{1}{2}\varepsilon_0 c E_0^2$$

$$\langle \vec{S} \rangle = \frac{1}{2}\varepsilon_0 c E_0^2\,\hat{z}$$

Momentum Density

$$\vec{\wp}_{em} = \mu_0 \epsilon_0 \vec{S} = \frac{\vec{S}}{c^2}$$

$$\langle \vec{\wp}_{em} \rangle = \frac{1}{c^2} \langle \vec{S} \rangle = \frac{1}{2} \frac{\epsilon_0 E_0^2}{c} \hat{z} = \frac{I}{c^2} \hat{z}$$

If the light falls on a surface and is absorbed, a volume $A c \Delta t$ of momentum is absorbed in time Δt. The force on area A is then

$$\vec{F} = \frac{\langle \vec{\wp}_{em} \rangle A c \Delta t}{\Delta t}$$

and the pressure exerted

$$P = \frac{F}{A} = \underline{\langle \vec{\wp}_{em} \rangle} \cdot c$$

$$= \frac{I}{c}$$

<u>Radiation Pressure</u> - Force per unit area exerted by EM wave

$$P_r = \frac{I}{c} \quad \text{absorbed}$$

$$= \frac{2I}{c} \quad \text{reflected}$$