EM Waves in Matter

If there are no free charges or currents Maxwell's equations in matter are

$$\nabla \cdot \vec{D} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

If the matter is a linear material (a big assumption)

$$\vec{D} = \varepsilon \vec{E} = \varepsilon_r \varepsilon_0 \vec{E}$$

$$\vec{B} = \mu \vec{H} = \mu_r \mu_0 \vec{H}$$

and Maxwell's equations become

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu \varepsilon \frac{\partial \vec{E}}{\partial t} = \mu_r \mu_0 \varepsilon_r \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Exactly the same as in vacuum with

$$\mu_0 \to \mu$$

$$\varepsilon_0 \to \varepsilon$$

The same method will yield the same wave equations with velocity c_m instead of c.

$$c_m = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \varepsilon_r \varepsilon_0}}$$

Dfn - Index of Refraction (n) The ratio of the speed of light in a material to the speed in vacuum.

$$n = \frac{c}{c_m} = \frac{\sqrt{\varepsilon \mu}}{\sqrt{\varepsilon_0 \mu_0}} = \sqrt{\varepsilon_r \mu_r}$$

$$\geq 1$$

$n = 1$ vacuum ~ 1 air

Everything we did up to now carries forward with $\mu_0 \rightarrow \mu$, $\varepsilon_0 \rightarrow \varepsilon$, $c \rightarrow c_m$

Energy Density (u)

$$u = \frac{1}{2} \varepsilon E^2 + \frac{1}{2\mu} B^2$$

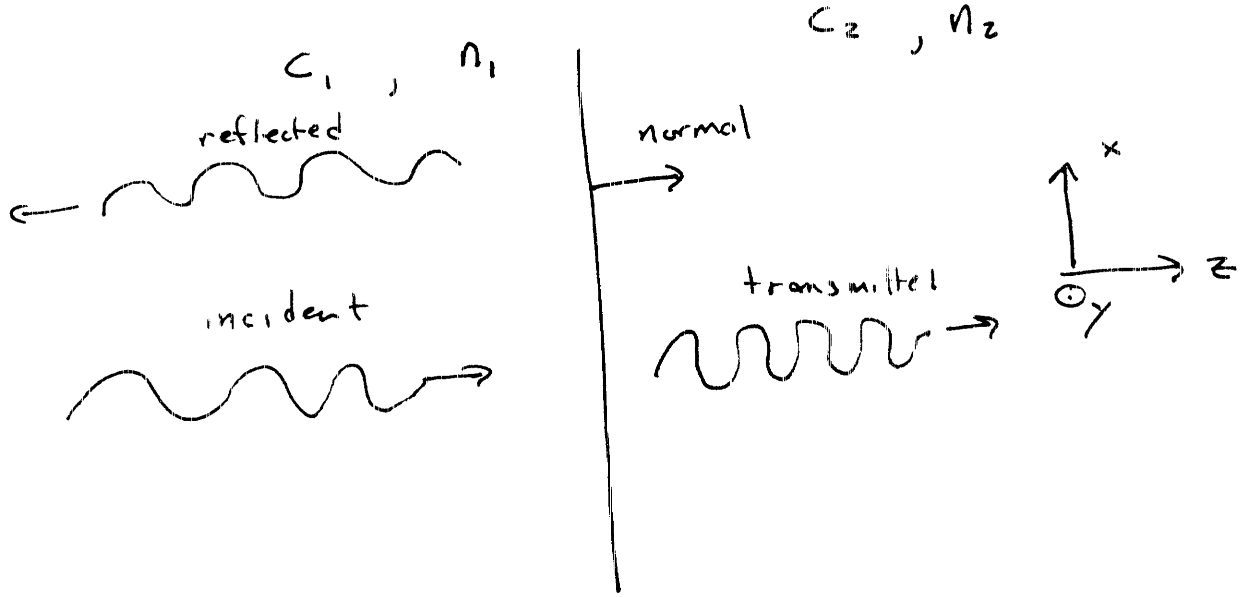
Poynting Vector - Energy flux

$$\vec{S} = \frac{1}{\mu} \vec{E} \times \vec{B}$$

Intensity

$$I = \frac{1}{2} c_m \varepsilon E_0^2$$

Now, we want to reflect a plane wave off a material surface and determine the reflected and transmitted waves.



Select z as $\hat{K} = \hat{z}$ the propagation direction and choose $\hat{n} = \hat{x}$ as the polarization direction.

The magnetic field direction is given by

$$\hat{K} \times \hat{n} = \hat{z} \times \hat{x} = \hat{y}$$

Incident Wave

$$\vec{E}_i = E_{oi}\, e^{i(k_1 z - \omega t)}\, \hat{x}$$

$$\vec{B}_i = B_{oi}\, e^{i(k_1 z - \omega t)}\, \hat{y} = \frac{E_{oi}}{c_1} e^{i(k_1 z - \omega t)}\, \hat{y}$$

$$c_1 = \frac{\omega}{k_1}$$

Reflected Wave

$$\vec{E}_r = E_{or}\, e^{i(-k_1 z - \omega t)}\, \hat{x}$$

$$\vec{B}_r = -\frac{E_{or}}{c_1} e^{i(-k_1 z - \omega t)}\, \hat{y}$$

where E_{or} may be complex.

Transmitted Wave

$$c_2 = \omega / k_2$$

$$\vec{E}_t = E_{ot}\, e^{i(k_2 z - \omega t)}\, \hat{x}$$

$$\vec{B}_t = \frac{E_{ot}}{c_2} e^{i(k_2 z - \omega t)}\, \hat{y}$$

E_{ot} may be complex.

Note, change in sign on reflected wave to give correct propagation direction from $\vec{S}$

Boundary Conditions - Go back to Maxwell

(1) $\varepsilon_1 \vec{E}_1^{\perp} = \varepsilon_2 \vec{E}_2^{\perp}$ (2) $\vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$

(3) $B_1^{\perp} = B_2^{\perp}$ (4) $\frac{1}{\mu_1} \vec{B}_1^{\parallel} = \frac{1}{\mu_2} \vec{B}_2^{\parallel}$

Note, $\varepsilon_1 = \varepsilon_{r_1} \varepsilon_0$ $\mu_1 = \mu_{r_1} \mu_0$

Let the interface be $z=0$ and we have already assumed the frequencies are the same to maintain continuity over time.

Begin applying BC at $z=0$.

(1) $\varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp}$

~~$\varepsilon_1 E_{\dots} = \varepsilon_1 E_{\dots}$~~

The $\perp$ direction is the $\hat{z}$ direction and none of the fields have a z component so this is trivially satisfied.

(3) Likewise $(B_1^{\perp} = B_2^{\perp} = 0)$

(2) $\vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$ - Components tangent to the interface.

$E_{i0} + E_{r0} = E_{t0}$

(4) $$\frac{1}{\mu_1}\vec{B}_1'' = \frac{1}{\mu_2}\vec{B}_2''$$

$$\frac{1}{\mu_1}\left(\frac{E_{0i}}{c_1} - \frac{E_{0r}}{c_1}\right) = \frac{1}{\mu_2}\frac{E_{0t}}{c_2}$$

$$E_{0i} - E_{0r} = \frac{\mu_1}{\mu_2}\cdot\frac{c_1}{c_2}\, E_{0t}$$

Introduce $\beta = \dfrac{\mu_1 c_1}{\mu_2 c_2} = \dfrac{\mu_1 n_2}{\mu_2 n_1} \approx \dfrac{n_2}{n_1}$ if $\mu_1 \approx \mu_2$ which is true for most conductors.

$$E_{0i} - E_{0r} = \beta E_{0t} \qquad (1)$$

$$E_{0i} + E_{0r} = E_{0t} \qquad (2)$$

Very like reflection of string.

(1) + (2) $\Rightarrow$ $2E_{0i} = (1+\beta)E_{0t}$

(3) $$\frac{E_{0t}}{E_{0i}} = \frac{2}{1+\beta}$$ real $\Rightarrow$ no phase shift on transmission

(2) + (3)

$$E_{0i} + E_{0r} = \left(\frac{2}{1+\beta}\right)E_{0i}$$

$$\frac{E_{0r}}{E_{0i}} = \frac{2}{1+\beta} - 1 = \frac{1-\beta}{1+\beta}$$

No phase shift if $1-\beta > 0$, phase shift of π if $1-\beta < 0$

If $1-\beta < 0$,

$$\left|\frac{E_{or}}{E_{oi}}\right| = -\left|\frac{1-\beta}{1+\beta}\right| = \left|\frac{1-\beta}{1+\beta}\right| e^{i\pi}$$

In terms of n_1, n_2 (if $\mu_1 \approx \mu_2$)

$$\frac{E_{or}}{E_{oi}} = \frac{n_1 - n_2}{n_1 + n_2} \qquad \frac{E_{ot}}{E_{oi}} = \frac{2n_1}{n_1 + n_2}$$

No phase shift in reflected wave if $n_1 > n_2$, phase shift of π if $n_1 < n_2$.

We would really like to know how much energy is reflected and transmitted. The intensity is given by

$$I = \frac{1}{2} c_m \varepsilon E_o^2$$

<u>Define Reflection Coefficient</u>

$$R \equiv \frac{I_r}{I_o} = \frac{E_{or}^* E_{or}}{E_{oi}^* E_{oi}} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

<u>Transmission Coefficient</u>

$$T = \frac{I_t}{I_o} = \frac{E_{ot}^2}{E_{oi}^2} = \frac{4n_1 n_2}{(n_1+n_2)^2}$$

$$\boxed{R+T=1}$$