Electrostatics

Suppose no charge is moving, $\vec{J} = 0$ everywhere.

$$\frac{\partial \vec{B}}{\partial t} = 0 \qquad \frac{\partial \vec{E}}{\partial t} = 0$$

Maxwells Eqns Become

$$\nabla \cdot \vec{E} = \rho/\epsilon_0$$

$$\nabla \times \vec{E} = 0$$

$$\left.\begin{aligned} \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{B} &= 0 \end{aligned}\right\rangle \Rightarrow \vec{B} = 0$$

A function is uniquely determined by its div and curl, $\vec{B} = 0$ satisfy $\nabla \cdot \vec{B} = \nabla \times \vec{B} = 0$, so $\vec{B} = 0$.

Since $\nabla \times \vec{E} = 0$, $\exists$ a function V s.t.

$$\vec{E} = -\nabla V$$

~~Call V the~~

Scalar Potential (V) - Function s.t.

$$\vec{E} = -\nabla V$$

- Units Volts $1\,V = 1\,\frac{Nm}{C} = 1\,\frac{J}{C}$
- Only defined up to a constant
- Do not confuse with potential energy U.

Poisson's Eqn

$$\nabla\cdot\vec{E} = \rho/\varepsilon_0 \implies \nabla\cdot\nabla V = -\rho/\varepsilon_0$$

$$\implies \nabla^2 V = -\rho/\varepsilon_0$$

In charge free regions,

Laplace's Eqn

$$\nabla^2 V = 0$$

Work - The work done by an external agent to move a particle from point A to point B along some path is

$$W = \int_{\vec{r}_A \to \vec{r}_B} \vec{F}_{ext} \cdot d\vec{l}$$

- $\vec{F}_{ext}$ force exerted by external agent
- $d\vec{l}$ is an element of the path

If the external agent moves the particle s.t. its kinetic energy does not change by slightly overcoming the resisting force of the system $\vec{F}$.

$$\vec{F}_{ext} = -\vec{F}$$

$$W = -\int_{\vec{r}_A \to \vec{r}_B} \vec{F} \cdot d\vec{l}$$

If the resisting force is supplied by an electric field, the Lorentz force gives

$$\vec{F} = q\vec{E}$$

$$W = -q \int_{\vec{r}_A \to \vec{r}_B} \vec{E} \cdot d\vec{l}$$

The electric force is conservative, $\nabla \times \vec{F} = 0$, so this work is the change in potential energy of the system between points A and B.

Potential Energy (U) – Units Joules

$$W = -q \int_{\vec{r}_A \to \vec{r}_B} \vec{E} \cdot d\vec{l} = U(\vec{r}_B) - U(\vec{r}_A)$$

, if $\Delta K = 0$.

Use $\vec{E} = -\nabla V$

$$W = q \int_{\vec{r}_A \to \vec{r}_B} \nabla V \cdot d\vec{l} = q \int_{\vec{r}_A \to \vec{r}_B} dV$$

$$= q\left(V(\vec{r}_B) - V(\vec{r}_A) \right)$$

gradient thm

$$W = U(\vec{r}_B) - U(\vec{r}_A) = qV(\vec{r}_B) - qV(\vec{r}_A)$$

Electric Potential Difference (ΔV_{AB})

$$\Delta V_{AB} = V(\vec{r}_B) - V(\vec{r}_A)$$

= Work per unit charge to move a charge q from $A \rightarrow B$.

The electric potential is only defined up to a constant, so we are allow to adjust V by adding a constant.

We can eliminate this uncertainty by arbitrarily chosing V at some point $\vec{r}_0$; $V(\vec{r}_0) = C$ where $\vec{r}_0$ is called the reference point. For point charges, the traditional choice is $V(\infty) = 0$.

The potential $V(\vec{r}_A)$ is then the potential difference with respect to the reference point

$$V(\vec{r}_A) = \Delta V_{0A}$$

the work per charge to move a charge from the reference point to A.

Principle of Linear Superposition

The total field is the vector sum of the field of individual charges.

$\Rightarrow$ The potential is additive

Suppose the charge can be divided into two pieces

$$\rho = \rho_1 + \rho_2$$

The field of one piece, ignoring the other piece can be found by solving

$$\nabla \cdot \vec{E}_1 = \rho_1/\epsilon_0 \qquad \nabla \cdot \vec{E}_2 = \rho_2/\epsilon_0$$

$$\nabla \times \vec{E}_1 = 0 \qquad \nabla \times \vec{E}_2 = 0$$

– or –

$$\nabla^2 V_1 = -\rho_1/\epsilon_0 \qquad \nabla^2 V_2 = -\rho_2/\epsilon_0$$

The field of the total distribution solve

$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \qquad \text{– or –} \qquad \nabla^2 V = -\rho/\epsilon_0$$

$$\nabla \times \vec{E} = 0$$

Evidently $\vec{E} = \vec{E}_1 + \vec{E}_2$ or $V = V_1 + V_2$

solve the full field.