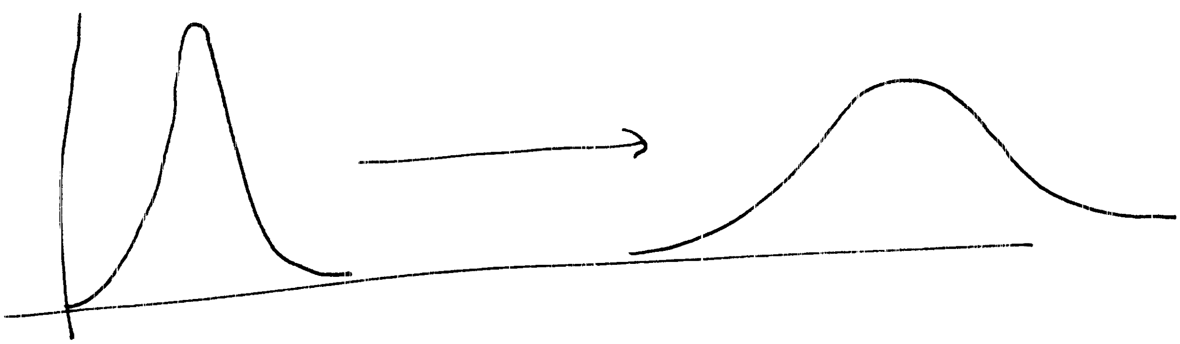
Attenuation + Dispersion

In real media, not all waves travel at the same speed, $\Rightarrow$ index of refraction is frequency dependent, $n(\omega)$.

Since different colors (frequencies) travel at different speeds, wave packets composed of multiple frequencies change shape as they travel.



This effect of different colors travelling different speeds is called <u>dispersion</u>.

The speed of any specific frequency is called the phase velocity, $c(\omega) = \dfrac{c}{n(\omega)} = \dfrac{\omega}{k}$

The speed of a pulse $(A(k))$, what is physically observed is called the group velocity, v_g,

$$v_g = \frac{d\omega}{dk}$$

<u>Attenuation</u> - As a wave travels through any medium, energy is transferred from the wave to the medium causing a decrease in amplitude of wave.

We can capture the attenuation of a dielectric media by allowing the susceptibity to become complex, $\tilde{\chi}_e$

$$\vec{P} = \varepsilon_0 \tilde{\chi}_e \vec{E}$$

where $\vec{E}$ was already complex.

We than have a complex $\vec{D}$ field,

$$\vec{D} = \vec{P} + \varepsilon_0 \vec{E} = \varepsilon_0 (1 + \tilde{\chi}_e) \vec{E}$$

<u>Complex Permittivity</u>

$$\tilde{\varepsilon} = \varepsilon_0 (1 + \tilde{\chi}_e)$$

Wave equation in dispersive medium,

$$\nabla^2 \vec{E} = \tilde{\varepsilon} \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

with wave equation

$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)} \qquad k \text{ complex}$$

$$(ik)^2 = \tilde{\varepsilon}\mu_0 (-i\omega)^2$$

$$k^2 = \tilde{\varepsilon}\mu_0 \omega^2$$

$$k = \omega\sqrt{\tilde{\varepsilon}\mu_0} = k_r + i k_i \quad \text{as before}$$

$$\vec{E} = E_0 e^{-k_i z} e^{i(k_r z - \omega t)}$$

<u>Attenuation (of intensity) coefficient</u>

$$\alpha = 2 k_i$$

<u>Wave Speed</u>

$$c_m = \frac{\omega}{k_r}$$

<u>Index of Refraction</u> $\quad n = \dfrac{c}{c_m} = \dfrac{c k_r}{\omega}$