Conduction

The electromagnetic field in a conductor exerts a force on the mobile charge to create a current.

Under the assumption of linearity,

$$\vec{J} = \sigma\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Under most circumstances, the $\vec{E}$ term is dominant.

Ohm's Law

$$\vec{J} = \sigma \vec{E}$$

Conductivity (σ) - Intrinsic property of conductors controlling the amount of current.

- Units $[\sigma] = \frac{S}{m}$ $\quad S \equiv$ Sieman
- $1\,S = \frac{1}{\Omega}$
- Ohm $1\,\Omega = 1\,V/A$
- Archaic $1\,S = 1$ mho $= 1\,\mho$

Resistivity (ρ) - Intrinsic property that resists current flow.

$$\rho = \frac{1}{\sigma}$$

- $[\rho] = \Omega \cdot m$

Resistance (R)

$$R = \frac{\Delta V}{I}$$

$$[R] = \Omega$$

Power (P) - Power transferred from field to current

$$P = \vec{F} \cdot \vec{v} = Q \vec{E} \cdot \vec{v}$$

velocity

Power Per Unit Volume

$$\frac{dP}{dV} = \frac{Q}{V} \vec{E} \cdot \vec{v} = \rho \vec{E} \cdot \vec{v}$$

charge density

$$\vec{J} = \rho \vec{v}$$

$$\frac{dP}{dv} = \vec{E} \cdot \vec{J}$$

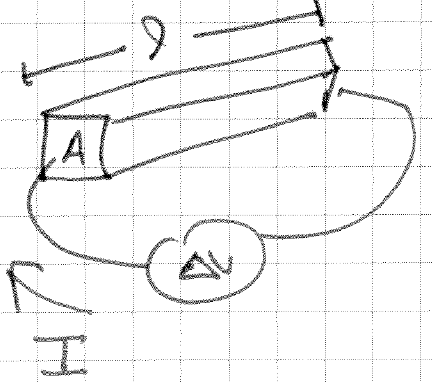
$$= \sigma \vec{E}^2 = \frac{\vec{J}^2}{\sigma}$$

This power is converted into thermal energy in the material.

<u>Dfn Joule Heating</u> Power dissipated as heat in a conductor.

$$\frac{dP}{dV} = \vec{E} \cdot \vec{J}$$

<u>Ex</u> Consider a rectalinear block of material with cross-sectional area A and length l. A potential difference ΔV is established across ends causing a current I to flow.



$$R = \frac{\Delta V}{I}$$

Assume a uniform current down the bar, $\vec{J} = J_0 \hat{x}$.

By Ohm's Law, $\vec{E} = \vec{J}/\sigma = \frac{J_0}{\sigma} \hat{x}$

$\Rightarrow$ Note, if we know the current we know the field.

The potential difference is

$$|\Delta V| = \left| - \int \vec{E} \cdot d\vec{l} \right| = \frac{J_0}{\sigma} l$$

The total current is

$$I = \int J \, da = J_0 A$$

Resistance

$$R = \frac{\Delta V}{I} = \frac{\frac{J_0 l}{\sigma}}{J_0 A} = \frac{1}{\sigma} \frac{l}{A}$$

$$= \rho \frac{l}{A}$$

Power converted into thermal energy

$$P = \left(\frac{dP}{dV}\right) V = \frac{dP}{dV} A l$$

power per volume

volume

$$P = \left(\frac{J_0^2}{\sigma}\right) A l = \left(\frac{J_0 l}{\sigma}\right) \cdot J_0 A$$

$$= \Delta V \, I$$

Power Dissipated in Electric Device

$$P = I \Delta V$$

	ρ (20°C)
Copper	$1.7 \times 10^{-8}\ \Omega m$
Graphite	$1.4 \times 10^{-5}\ \Omega m$
Salt water	$4.4 \times 10^{-2}\ \Omega m$
Pure Water	$2.5 \times 10^{5}\ \Omega m$
Glass	$10^{10} - 10^{14}\ \Omega m$

Ex Now suppose conductivity changes along the bar, perhaps because of a temperature gradient.

$$\sigma(x) = \gamma(x + x_0)$$

Sln Again assume steady current $\vec{J} = J_0 \hat{x}$ otherwise a net charge builds up in the material.

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{J_0}{\gamma(x+x_0)} \hat{x}$$

Potential Difference

$$\Delta V = -\int E \cdot d\vec{l}$$

$$= -\int_0^l \frac{J_0}{\gamma(x+x_0)} dx$$

$$= -\frac{J_0}{\gamma} \ln\left(\frac{l+x_0}{x_0}\right)$$

Total Current

$$I = J_0 A$$

Resistance

$$R = \frac{|\Delta V|}{I} = \frac{1}{\gamma A} \ln\left(\frac{l + x_0}{x_0} \right)$$

$\Rightarrow$ Since $\vec{E}$ changes with position we do devolop a static charge density as the current is set up

$$\nabla \cdot \vec{E} = \rho / \varepsilon_0$$

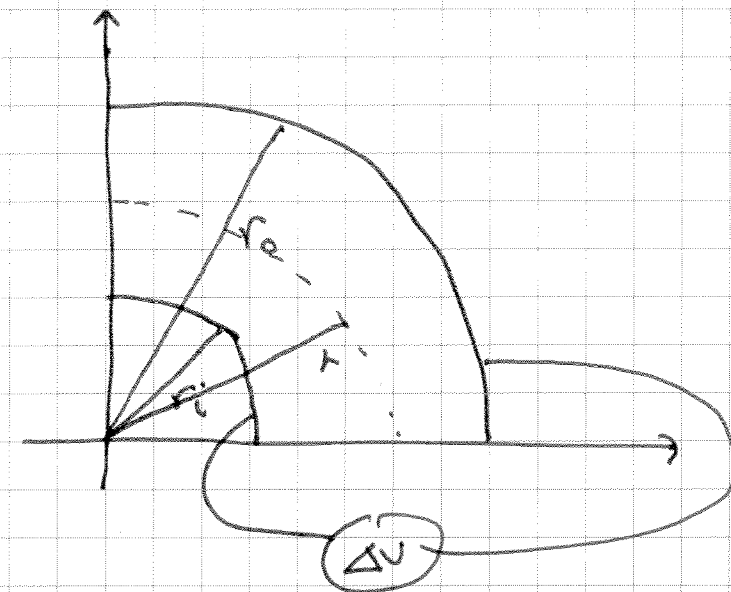
$$\rho = \varepsilon_0 \nabla \cdot \vec{E}$$

$$= \varepsilon_0 \frac{d}{dx} \frac{J_0}{\gamma (x + x_0)}$$

$$= - \frac{J_0 \varepsilon_0}{\gamma (x + x_0)^2}$$

Ex Consider the resistance of a cylindrical ~~wedge~~ wedge of thickness a, inner radius r_i and outer radius r_o

Thickness a into page.



The total current through any cross-section must be constant or charge progessively accumulates.

The area of a cross section is

$$A(s) = \frac{\pi}{2} s a$$

Let I be the total current provided by the supply

$$I = J_i A(r_i) = J_o A(r_o)$$

The current density is then

$$\vec{J} = \frac{I}{A(s)}\,\hat{s} = \frac{2I}{\pi s a}\,\hat{s}$$

The field is then by Ohm's Law

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{I}{\sigma A(s)}\,\hat{s}$$

$$= \frac{2I}{\sigma \pi a s}\,\hat{s}$$

<u>Potential Difference</u>

$$\Delta V = -\int \vec{E}\cdot d\vec{l} = -\int_{r_i}^{r_o} \frac{2I}{\sigma \pi a s}\,ds$$

$$= -\frac{2I}{\sigma \pi a} \ln\left(\frac{r_o}{r_i}\right)$$

<u>Resistance</u>

$$R = \frac{\Delta V}{I} = \frac{2}{\sigma \pi a} \ln\left(\frac{r_o}{r_i}\right)$$

Power Dissipated in Wedge

$$\text{Power} = P = \int_V \frac{dP}{dV}\, d\tau$$

$$= a\int_0^{\pi/2} d\phi \int_{r_i}^{r_o} s\,ds\, \frac{dP}{dV} \qquad d\tau = a(ds)(s\,d\phi)$$

$$= a\int_0^{\pi/2} d\phi \int_{r_i}^{r_o} s\,ds \left(\frac{I^2}{\sigma A^2}\right)$$

$$\frac{dP}{dV} = \vec{E}\cdot\vec{J} = \frac{J^2}{\sigma} = \frac{I^2}{\sigma A^2}$$

$$P = a\int_0^{\pi/2} d\phi \int_{r_i}^{r_o} ds\; s \left(\frac{I^2}{\sigma\left(\frac{\pi}{2} s a\right)^2}\right)$$

$$= \frac{2}{\pi}\,\frac{1}{\sigma a}\, I^2 \int_{r_i}^{r_o} \frac{ds}{s}$$

$$= \frac{2I^2}{\pi a \sigma} \ln\left(\frac{r_o}{r_i}\right) = I^2 R$$