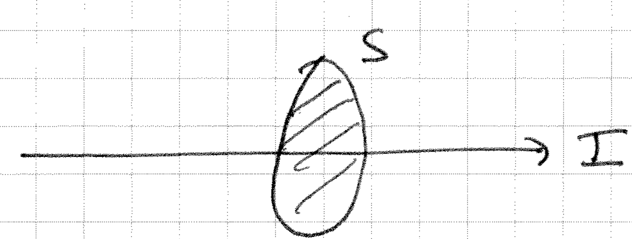
# Electric Current

Current ($I$) - Charge per unit time flowing through surface S.



$\Rightarrow$ Units Amperes = Amps $\quad 1A = 1\frac{C}{s}$

$\Rightarrow$ Vector Current $\vec{I} = I\hat{n}$ where $\hat{n}$ points in the direction of current flow

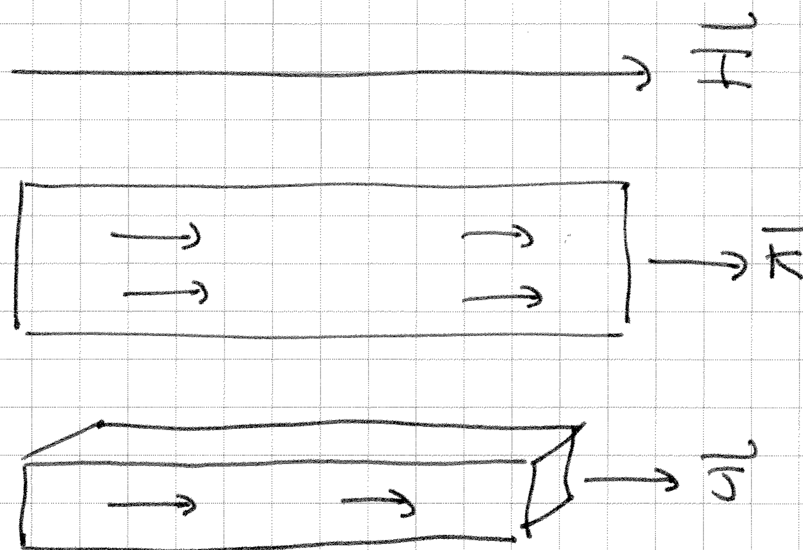
$\Rightarrow$ Positive current flow is in the direction of the flow of positive charge.

Current Density ($\vec{J}$) - The charge per unit area per unit time flowing in same region

$$\vec{J} = \frac{dI}{da}\hat{n} = \frac{d\vec{I}}{da}$$

## Surface Current Density ($\vec{K}$) Current per unit length flowing in $\hat{n}$ direction

$$\vec{K} = \frac{dI}{dl}\hat{n}$$




## Total Current I through surface S

$$I = \int_S \vec{J} \cdot d\vec{a}$$

$$= \int_C \vec{K} \cdot \hat{n}\, dl$$

$\hat{n}$: normal to C

Law of Conservation of Charge (Local) The time rate of change of the net charge in a volume must equal the total flow of charge into the volume.

$$I_{in} = -\int \vec{J} \cdot \hat{n}\, da = \frac{d}{dt}\int_V \rho\, d\tau$$

outward normal

Divergence Thm

$$I_{in} = -\int_V \nabla \cdot \vec{J}\, d\tau = \frac{d}{dt}\int_V \rho\, d\tau$$

for all V

$$-\nabla \cdot \vec{J} = \frac{\partial \rho}{\partial t}$$

Continuity Eqn - Expresses conservation of charge

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Find continuity eqn in Maxwell's eqns

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Take divergence of Ampere's Law

$$\nabla \cdot (\nabla \times \vec{B}) = 0 = \mu_0 \nabla \cdot \vec{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \nabla \cdot \vec{E}$$

always

Use Gauss

$$\mu_0 \nabla \cdot \vec{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\rho/\varepsilon_0\right) = 0$$

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

We can create current densities by allowing charge densities to move with velocity $\vec{v}$.

$\vec{J} = \rho \vec{v}$ – volume charge density moves with velocity $\vec{v}$

$\vec{K} = \sigma \vec{v}$ – surface charge density $\sigma$ moves with velocity $\vec{v}$.

$\vec{I} = \lambda \vec{v}$ – linear charge density $\lambda$ moves with velocity $\vec{v}$.

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<u>Ex</u> <u>Griffiths Problem 5.6</u> – A thin disk with surface charge density $\sigma$ is rotated at angular velocity $\omega$. Compute $\vec{K}$

<u>Sln</u>

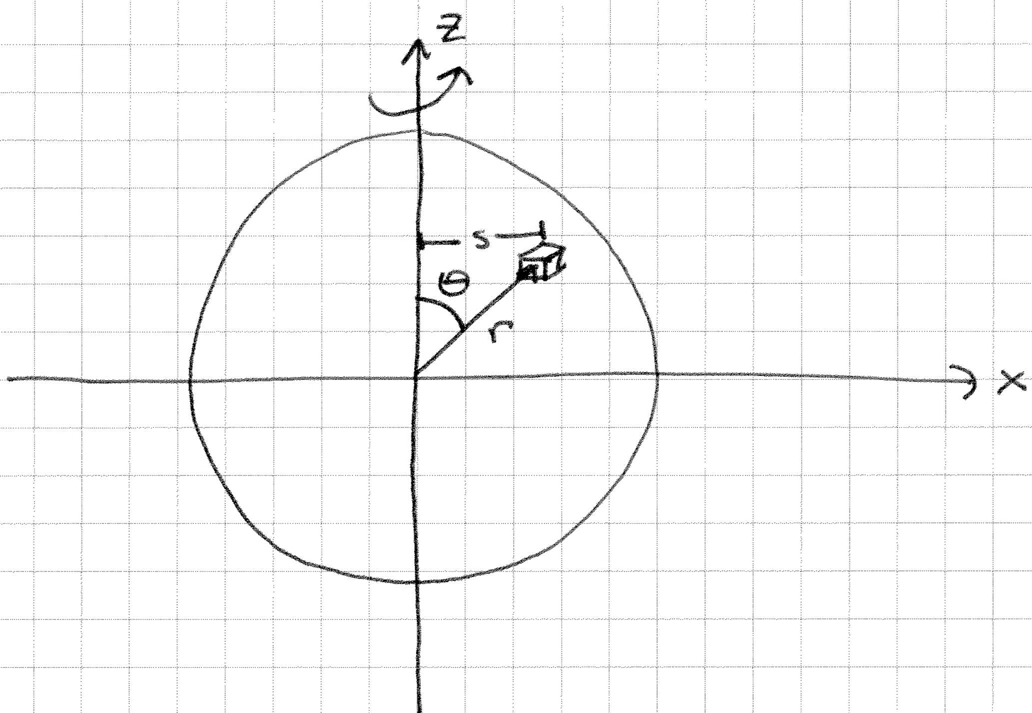
$$\vec{K} = \sigma \vec{v}$$

$$|\vec{v}| = s\omega \qquad \omega = \frac{d\phi}{dt}$$

$$\vec{v} = s\omega\hat{\phi} = s\frac{d\phi}{dt}\hat{\phi}$$

$$\vec{K} = \sigma s \omega \hat{\phi}$$

<u>Ex</u> A uniformly charged sphere with volume charge density $\rho$ is rotated with angular velocity $\omega$ about $z$ axis




$$s = r\sin\theta$$

$$v = s\omega = r\omega\sin\theta$$

$$\vec{J} = \rho\vec{v} = \rho r\omega\sin\theta\,\hat{\phi}$$

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Note, we can also write a current as

$$\vec{I} = \sum_i q\vec{v}_i ,$$ but this would not be a steady state current