Differential Eqn for Vector Potential

$$\nabla \times \vec{B} = \mu_0 \vec{J} = \nabla \times (\nabla \times \vec{A})$$

$$= \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

Gauge Choices

The electric potential is undefined up to a constant because the field is the measurable and $\vec{E} = -\nabla V$ kills the constant. For $\vec{A}$, the field is $\vec{B} = \nabla \times \vec{A}$ so $\vec{A}$ is undefined by any function whose curl is zero. The curl of a gradient is always zero, so we can always change $\vec{A}$ by $\nabla \lambda$ and get the same physics.

$$\vec{B} = \nabla \times \vec{A} = \nabla \times (\vec{A} + \nabla \lambda)$$

As such, we can pick λ so $\nabla \cdot \vec{A}' = 0$

$$\vec{A}' = \vec{A} + \nabla \lambda$$

$$\nabla \cdot \vec{A}' = \nabla \cdot \vec{A} + \nabla^2 \lambda$$

$$\nabla^2 \lambda = -\nabla \cdot \vec{A}$$

The choice $\nabla \cdot \vec{A} = 0$ is called a gauge choice.
This particular choice is called the Coulomb Gauge.

In the Coulomb Gauge,

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

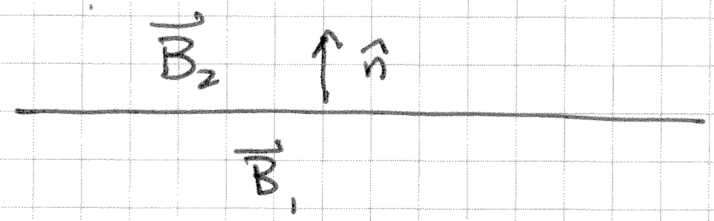
which is 3 copies of Poisson's eqn.

Magnetostatic Scalar Potential - In current free regions $\vec{J} = 0$, $\nabla \times \vec{B} = 0$, and we can introduce

$$\vec{B} = -\nabla V_m$$

where V_m is the magnetostatic scalar potential.
Since $\nabla \cdot \vec{B} = 0$, $\nabla^2 V_m = 0$ and we have Laplace's eqn.

Magnetostatic Boundary Conditions



$$\vec{B}_2 - \vec{B}_1 = \mu_0 (\vec{K} \times \hat{n})$$

Since $\vec{B} = \nabla \times \vec{A}$, $\vec{A}$ must be continuous at the interface, $\vec{A}_2 = \vec{A}_1$.

$\Rightarrow$ All derivatives of $\vec{A}$ along the interface must also be continuous.

Let $\hat{n} = \hat{z}$, then $\frac{\partial \vec{A}}{\partial x}$ and $\frac{\partial \vec{A}}{\partial y}$ is continuous.

Use the boundary condition on $\vec{B}$ to develop a boundary condition on $\vec{A}$

$$\vec{K} \times \hat{n} = (K_x, K_y, 0) \times \hat{z} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ K_x & K_y & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \hat{x} K_y - \hat{y} K_x$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{y}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)$$

$$+ \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\vec{B}_2 - \vec{B}_1 = \nabla \times A_2 - \nabla \times A_1$$

$$= \hat{x}\left(-\frac{\partial A_{y2}}{\partial z} + \frac{\partial A_{y1}}{\partial z}\right)$$

$$- \hat{y}\left(-\frac{\partial A_{x2}}{\partial z} + \frac{\partial A_{x1}}{\partial z}\right)$$

$$+ \hat{z}\,(0)$$

After dropping all $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$ because they cancel.

Equating components

$\hat{x}$: $$\frac{\partial A_{y2}}{\partial z} - \frac{\partial A_{y1}}{\partial z} = -\mu_0 K_x$$

$\hat{y}$: $$\frac{\partial A_{x2}}{\partial z} - \frac{\partial A_{x1}}{\partial z} = -\mu_0 K_y$$

$\hat{z}$: $0 = 0$

Generalizing to any interface

$$\frac{\partial \vec{A}_2}{\partial n} - \frac{\partial \vec{A}_1}{\partial n} = -\mu_0 \vec{K}$$