The "H" Field

The bound currents behave just as any other currents in producing magnetic fields.

Let $\vec{J}_f$ be the free currents, any current not produced by magnetization.

Ampere's Law

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J}_f + \vec{J}_b \right)$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} \right) - \vec{J}_b = \vec{J}_f$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} \right) - \nabla \times \vec{M} = \vec{J}_f$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_f$$

Define $$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

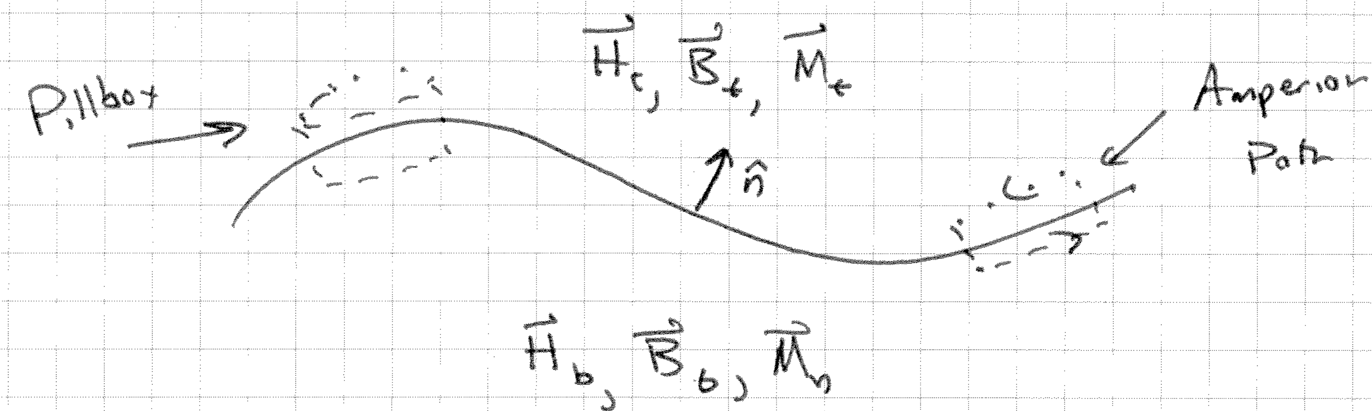
$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$$\nabla \times \vec{H} = \vec{J}_f$$

or

$$\oint_C \vec{H} \cdot d\vec{l} = I_{f,enc}$$

Boundary Conditions



$$\nabla \cdot \vec{B} = 0 \implies B_t^\perp = B_b^\perp$$

$$\implies H_t^\perp - H_b^\perp = -\left(M_t^\perp - M_b^\perp\right)$$

Amperian Path

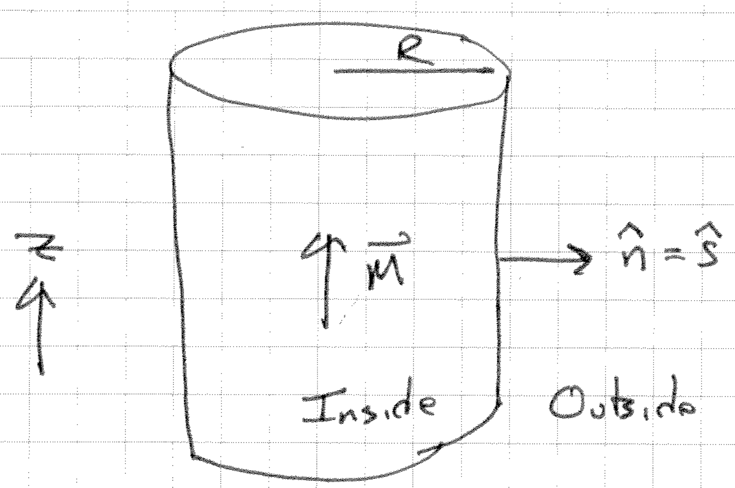
$$\oint_C \vec{H} \cdot d\vec{l} = I_{fenc}$$

$$\left| H_t^\parallel - H_b^\parallel \right| = \left| K_f \cdot \hat{t} \right|$$

free surface current

$$\vec{H}_t^{\parallel} - \vec{H}_b^{\parallel} = \vec{K}_f \times \hat{n}$$

Ex Infinite cylinder with radius R and magnetization $\vec{M} = M_0 \hat{z}$



No free current

$\vec{H} = 0$.

On the curved sides,

$$\underset{\overset{\shortparallel}{0}}{H_o^{\perp}} - H_i^{\perp} = -(\underset{\overset{\shortparallel}{0}}{M_2^{\perp}} - M_1^{\perp})$$

$$\underset{\overset{\shortparallel}{0}}{\vec{H}_o^{\parallel}} - \vec{H}_i^{\parallel} = \underset{\overset{\shortparallel}{0}}{\vec{K}_f} \times \hat{s}$$

Outside

$$\mu_0 \vec{H}_0 = \vec{B}_0 - \mu_0 \underset{0}{\underset{\shortparallel}{\vec{M}_0}} = 0$$

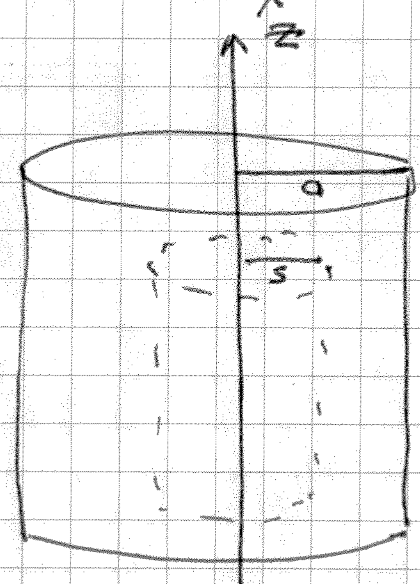
$$\Rightarrow \vec{B}_0 = 0$$

Inside

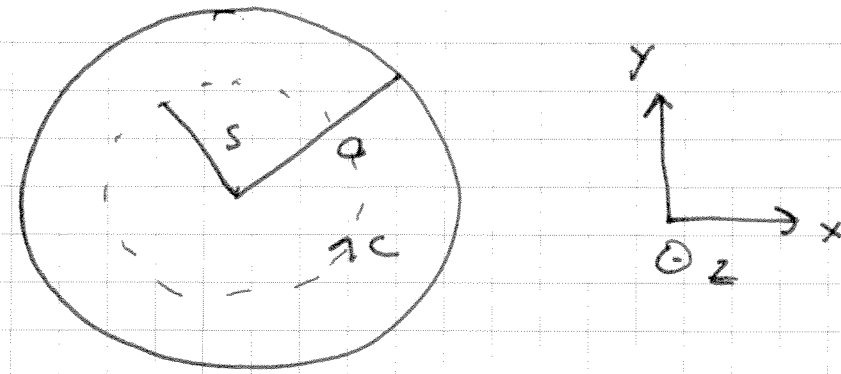
$$\mu_0 \vec{H}_i = 0 = \vec{B}_i - \mu_0 \vec{M}_i$$

$$\vec{B}_i = \mu_0 \vec{M}_i = \mu_0 M_0 \hat{z} \quad \checkmark$$

Ex A conductor of radius a carries a current $\vec{J} = \gamma s^2 \hat{z}$. Compute what you can.



End View



Positive normal out of page by RHR.

Inside

$$I_{f,enc} = \int_0^s \vec{J} \cdot d\vec{a} = \int_0^s J\, da$$

$$= \int_0^{2\pi} \int_0^s J\, s\, ds\, d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^s \gamma s^3\, ds$$

$$= 2\pi \cdot \frac{\gamma s^4}{4} = \frac{\pi \gamma s^4}{2}$$

$$\oint_c \vec{H} \cdot d\vec{l} = 2\pi s H_i = I_{fenc}$$

$$\vec{H}_i = \frac{I_{fenc}}{2\pi s} \text{ ccw}$$

RHR

$$\vec{H}_i = \frac{\pi \gamma s^4/2}{2\pi s} \text{ ccw}$$

$$= \frac{\gamma s^3}{4} \text{ ccw}$$

Outside Conductor $(s > a)$

$$I_{fenc} = \frac{\pi \gamma a^4}{2}$$

$$\vec{H}_o = \frac{I_{fenc}}{2\pi s} \text{ ccw}$$

$$= \frac{\pi \gamma a^4/2}{2\pi s} \text{ ccw} = \frac{\gamma a^4}{4s} \text{ ccw}$$

Outside Conductor $\vec{M}_o = 0$

$$\vec{H}_o = \frac{\vec{B}_o}{\mu_0} - \vec{M}_o = \frac{\vec{B}_o}{\mu_0} = \frac{\gamma a^4}{4s} \text{ ccw}$$

$$\vec{B}_o = \frac{\gamma a^4 \mu_0}{4s} \text{ ccw}$$

Inside Conductor $\vec{M}_i$ is unknown, so we cannot find $\vec{B}_i$.