Homework 1

Due Friday 1/25/2013 - at beginning of class

Reading Assignment - Chapter 1

Griffiths Problems

Each problem should be started on its own piece of paper. Points will be removed from solutions that are difficult to read.

1.5 (Note, this is a nice time to try a symbolic math package).

1.11

1.13

1.15

1.18

1.25

1.26

Problem E.1.1 Calculate ∇V where $V = \vec{p} \cdot \vec{r}/r^3$ where $\vec{p}$ is a constant vector. Note this is the potential of a point dipole.
Problem E.1.2 Consider the vector field $\vec{E} = \gamma \hat{z} \times \vec{r}$ where γ is a constant. Sketch the field. Compute the line integral of the field around a circle of radius R in the $x - y$ plane by direct integration. Compute the same integral using Stoke's Thm.
Problem E.1.3 Consider the vector field $\vec{E} = \gamma \vec{r}$ where γ is a constant. Sketch the field. Compute the flux, $\oint \vec{E} \cdot d\vec{a}$, out of the cube $0 < x < 1$, $0 < y < 1$, and $0 < z < 1$ by direct integration. Compute the flux using the Divergence Thm.
Problem E.1.4 Consider the function $f = x^2 + y^2 + z^2$. Compute the gradient in both Cartesian and spherical coordinates. Show the answers agree.

1.5 Show $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$= \hat{x}(B_y C_z - B_z C_y)$$

$$- \hat{y}(B_x C_z - B_z C_x)$$

$$+ \hat{z}(B_x C_y - B_y C_x)$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_y C_z - B_z C_y & B_z C_x - B_x C_z & B_x C_y - B_y C_x \end{vmatrix}$$

x - component

$$A_y\left(\underset{①}{B_x C_y} - \underset{③}{B_y C_x}\right) - A_z\left(\overset{④}{B_z C_x} - \overset{②}{B_x C_z}\right)$$

Work out $\vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

$$= \vec{B}\left(A_x C_x + A_y C_y + A_z C_z\right) - \vec{C}\left(A_x B_x + A_y B_y + A_z B_z\right)$$

x - component

$$\underline{B_x A_x C_x} + \overset{①}{B_x A_y C_y} + \overset{②}{B_x A_z C_z}$$

$$- \underline{C_x A_x B_x} - \underset{③}{C_x A_y B_y} - \underset{④}{C_x A_z B_z}$$

Underlined terms cancel, ⓘ matches terms with x - component of $\vec{A}\times(\vec{B}\times\vec{C})$

That was boring. I worked out the full result with Maple as follows.

> $with(LinearAlgebra);$

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations, GenerateMatrix, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, LA_Main, LUDecomposition, LeastSquares, LinearSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SubMatrix, SubVector, SumBasis, SylvesterMatrix, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip] (1)

> $assume(Ax, real);$

> $assume(Ay, real);$

> $assume(Az, real);$

> $assume(Bx, real);$

> $assume(By, real);$

> $assume(Bz, real);$

> $assume(Cx, real);$

> $assume(Cy, real);$

> $assume(Cz, real);$

> $A := < Ax, Ay, Az >;$

$$A := \begin{bmatrix} Ax\sim \\ Ay\sim \\ Az\sim \end{bmatrix} \tag{2}$$

> $B := < Bx, By, Bz >;$

$$B := \begin{bmatrix} Bx\sim \\ By\sim \\ Bz\sim \end{bmatrix} \tag{3}$$

$> C := < Cx, Cy, Cz >;$

$$C := \begin{bmatrix} Cx\sim \\ Cy\sim \\ Cz\sim \end{bmatrix} \tag{4}$$

$> left := CrossProduct(A, CrossProduct(B, C));$

$$left := \begin{bmatrix} Ay\sim\ (Bx\sim\ Cy\sim - By\sim\ Cx\sim) - Az\sim\ (Bz\sim\ Cx\sim - Bx\sim\ Cz\sim) \\ Az\sim\ (By\sim\ Cz\sim - Bz\sim\ Cy\sim) - Ax\sim\ (Bx\sim\ Cy\sim - By\sim\ Cx\sim) \\ Ax\sim\ (Bz\sim\ Cx\sim - Bx\sim\ Cz\sim) - Ay\sim\ (By\sim\ Cz\sim - Bz\sim\ Cy\sim) \end{bmatrix} \tag{5}$$

$> right := B \cdot DotProduct(A, C) - C \cdot DotProduct(A, B);$

$$right := \begin{bmatrix} (Ax\sim\ Cx\sim + Ay\sim\ Cy\sim + Az\sim\ Cz\sim)\ Bx\sim - (Ax\sim\ Bx\sim + Ay\sim\ By\sim + Az\sim\ Bz\sim)\ Cx\sim \\ (Ax\sim\ Cx\sim + Ay\sim\ Cy\sim + Az\sim\ Cz\sim)\ By\sim - (Ax\sim\ Bx\sim + Ay\sim\ By\sim + Az\sim\ Bz\sim)\ Cy\sim \\ (Ax\sim\ Cx\sim + Ay\sim\ Cy\sim + Az\sim\ Cz\sim)\ Bz\sim - (Ax\sim\ Bx\sim + Ay\sim\ By\sim + Az\sim\ Bz\sim)\ Cz\sim \end{bmatrix} \tag{6}$$

$> left - right;$

$$\Big[\ [\tag{7}$$

$$Ay\sim\ (Bx\sim\ Cy\sim - By\sim\ Cx\sim) - Az\sim\ (Bz\sim\ Cx\sim - Bx\sim\ Cz\sim) - (Ax\sim\ Cx\sim + Ay\sim\ Cy\sim + Az\sim\ Cz\sim)\ Bx\sim + (Ax\sim\ Bx\sim + Ay\sim\ By\sim + Az\sim\ Bz\sim)\ Cx\sim],\ [$$
$$Az\sim\ (By\sim\ Cz\sim - Bz\sim\ Cy\sim) - Ax\sim\ (Bx\sim\ Cy\sim - By\sim\ Cx\sim) - (Ax\sim\ Cx\sim + Ay\sim\ Cy\sim + Az\sim\ Cz\sim)\ By\sim + (Ax\sim\ Bx\sim + Ay\sim\ By\sim + Az\sim\ Bz\sim)\ Cy\sim],\ [$$
$$Ax\sim\ (Bz\sim\ Cx\sim - Bx\sim\ Cz\sim) - Ay\sim\ (By\sim\ Cz\sim - Bz\sim\ Cy\sim) - (Ax\sim\ Cx\sim + Ay\sim\ Cy\sim + Az\sim\ Cz\sim)\ Bz\sim + (Ax\sim\ Bx\sim + Ay\sim\ By\sim + Az\sim\ Bz\sim)\ Cz\sim]\ \Big]$$

$> simplify(\%);$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

$>$

1.11

(a) $\nabla(x^2+y^3+z^4)$

$= (2x, 3y^2, 4z^3)$

(b) $\nabla(x^2y^3z^4) = (2xy^3z^4, 3x^2y^2z^4, 4x^2y^3z^3)$

(c) $\nabla(e^x \sin y \ln z)$

$= \left(e^x \sin y \ln z, e^x \cos y \ln z, \frac{e^x \sin y}{z}\right)$

(1.13) $\vec{r}'' = \vec{r} - \vec{r}'$

$= (x - x', y - y', z - z')$

(a)

$\vec{r}''^2 = \vec{r}'' \cdot \vec{r}'' = (x - x')^2 + (y - y')^2 + (z - z')^2$

$\nabla r''^2 = \left(\frac{\partial}{\partial x} r'', \frac{\partial}{\partial y} r'', \frac{\partial}{\partial z} r'' \right)$

$= \left(2(x - x'), 2(y - y'), 2(z - z') \right)$

$= 2 \vec{r}''$

(b) $\nabla \left(\frac{1}{r''} \right) = \nabla \frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}$

$= -\frac{1}{2} \frac{1}{((x-x')^2 + (y-y')^2 + (z-z')^2)^{3/2}} \times$

$\left(2(x - x'), 2(y - y'), 2(z - z') \right)$

$$\nabla\left(\frac{1}{r''}\right) = \frac{-\vec{r}''}{r''^3} = \frac{-\hat{r}''}{r''^2}$$

(c) $$\nabla r''^n = \nabla\left(\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}\right)^n$$

$$= \left(\frac{n}{2}\right)\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{\left(\frac{n}{2}-1\right)}$$

$$\cdot\, 2\,\vec{r}''$$

$$= n\, r''^{(n-2)}\, r''\, \hat{r}''$$

$$= n\, r''^{(n-1)}\, \hat{r}''$$

1.15

(a) $\nabla \cdot (x^2, 3xz^2, -2xz)$

$= 2x + 0 - 2x = 0$

(b) $\nabla \cdot (xy, 2yz, 3zx)$

$= y + 2z + 3x$

(c) $\nabla \cdot (y^2, 2xy + z^2, 2yz)$

$= 0 + 2x + 2y$

1.18

(a) $\nabla \times \vec{v}_a = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 3xz^2 & -2xz \end{vmatrix}$

$$= \hat{x}(0 - 6xz)$$
$$- \hat{y}(-2x - 0)$$
$$+ \hat{z}(3z^2 - 0)$$

$$= (-6xz, +2x, +3z^2)$$

(b)

$\nabla \times \vec{v}_b = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3zx \end{vmatrix}$

$$\nabla \times \vec{v}_b = \hat{x}(0 - 2y)$$
$$- \hat{y}(3z - 0)$$
$$+ \hat{z}(0 - x)$$

$$= (-2y, -3z, -x)$$

(c)

$$\nabla \times \vec{v}_c = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & (2xy + z^2) & 2yz \end{vmatrix}$$

$$= \hat{x}(2z - 2z)$$
$$- \hat{y}(0 - 0)$$
$$+ \hat{z}(2y - 2y) = 0$$

1.25 (a)

Product role iv

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

$$\vec{A} = (x, 2y, 3z) \quad \vec{B} = (3y, -2x, 0)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & 2y & 3z \\ 3y & -2x & 0 \end{vmatrix}$$

$$= \hat{x}(0 + 6xz)$$

$$-\hat{y}(-9yz)$$

$$+\hat{z}(-2x^2 - 6y^2)$$

$$= (6xz, +9yz, -2x^2 - 6y^2)$$

$$\nabla \cdot (\vec{A} \times \vec{B}) = 6z + 9z = 15z$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & 2y & 3z \end{vmatrix}$$

$$= \hat{x}(0-0)$$
$$-\hat{y}(0-0)$$
$$+\hat{z}(0-0) \quad = 0$$

$$\nabla \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y & -2x & 0 \end{vmatrix}$$

$$= \hat{x}(0-0)$$
$$-\hat{y}(0-0)$$
$$+\hat{z}(-2-3) \quad = -5\hat{z}$$

$\vec{B} \cdot (\nabla \times \vec{A}) = 0$ $\qquad$ $\vec{A} \cdot (\nabla \times \vec{B}) = -15z$

$\vec{B} \cdot (\nabla \times \vec{A}) + \vec{A} \cdot (\nabla \times \vec{B}) = -15z$ ✓

(b) Product Rule ii

$$\nabla(\vec{A}\cdot\vec{B}) = \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A}) + (\vec{A}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{A}$$

$$\vec{A}\cdot\nabla = x\frac{\partial}{\partial x} + 2y\frac{\partial}{\partial y} + 3z\frac{\partial}{\partial z}$$

$$(\vec{A}\cdot\nabla)\vec{B} = (6y, -2x, 0)$$

$$(\vec{B}\cdot\nabla)\vec{A} = \left(3y\frac{\partial}{\partial x} - 2x\frac{\partial}{\partial y}\right)(x, 2y, 3z)$$

$$= (3y, -4x, 0)$$

We calculated $\nabla\times\vec{A}$ and $\nabla\times\vec{B}$ in the previous problem

$$\vec{A} \times (\nabla \times \vec{B}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & 2y & 3z \\ 0 & 0 & -5 \end{vmatrix}$$

$$= \hat{x}(-10y - 0)$$
$$-\hat{y}(-5x - 0)$$
$$+\hat{z}(0 - 0)$$

$$= (-10y, 5x, 0)$$

$$\vec{B} \times (\nabla \times \vec{A}) = 0$$

$$\vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) + (\vec{A} \cdot \nabla)\vec{B} + (\vec{B} \cdot \nabla)\vec{A}$$

$$= (-10y, 5x, 0) + 0 + (6y, -2x, 0)$$
$$+ (3y, -4x, 0)$$

$$= (-y, -x, 0)$$

$$\nabla(\vec{A}\cdot\vec{B}) = \nabla(3xy - 4xy)$$

$$= -\nabla(xy) = (-y, -x, 0) \quad \checkmark$$

(c) <u>Product Rule vi</u>

$$\nabla\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B}$$

$$+ \vec{A}(\nabla\cdot\vec{B}) - \vec{B}(\nabla\cdot\vec{A})$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\cdot\vec{A} = (1+2+3) = 6$$

$$\nabla\times(\vec{A}\times\vec{B}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xz & 9yz & -2x^2-6y^2 \end{vmatrix}$$

$$= \hat{x}(-12y - 9y)$$

$$-\hat{y}(-4x - 6x)$$

$$+\hat{z}(0 - 0) = (-21y, +10x, 0)$$

$$\vec{A}\,(\nabla\cdot\vec{B}) = 0$$

$$\vec{B}\,(\nabla\cdot\vec{A}) = (18y\ ,\ -12x,\ 0)$$

$$(\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B} + \vec{A}\,(\nabla\cdot\vec{B}) - \vec{B}\,(\nabla\cdot\vec{A})$$

$$= (3y\ ,-4x,0) - (6y\ ,-2x,0) + 0 - (18y,\ -12x,0)$$

$$= (-21y,\ 10x,\ 0)\ \checkmark$$

1.26

(a) $\nabla^2 T_a = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \left(x^2 + 2xy + 3z + 4 \right)$

$= 2$

(b) $\nabla^2 \left(\sin x \sin y \sin z \right)$

$= -3 \sin x \sin y \sin z$ (back to QM)

(c) $\nabla^2 \left(e^{-5x} \sin 4y \cos 3z \right)$

$$= \left(25 e^{-5x} \sin 4y \cos 3z - 16 e^{-5x} \sin 4y \cos 3z - 9 e^{-5x} \sin 4y \cos 3z \right) = 0$$

(d) $\nabla^2 \vec{v} = \left(\nabla^2 x^2, \nabla^2 3xz^2, -\nabla^2 2xz \right)$

$= \left(2, 6x, 0 \right)$

E.1.1

$$V = \frac{\vec{p} \cdot \vec{r}}{r^3}$$

$$= \frac{1}{r^3} \left(p_x x + p_y y + p_z z \right)$$

Product Rule i

$f = 1/r^3$
$g = \vec{p} \cdot \vec{r}$

$$\nabla fg = g \nabla f + f \nabla g$$

$$\nabla \left(p_x x + p_y y + p_z z \right) = \vec{p}$$

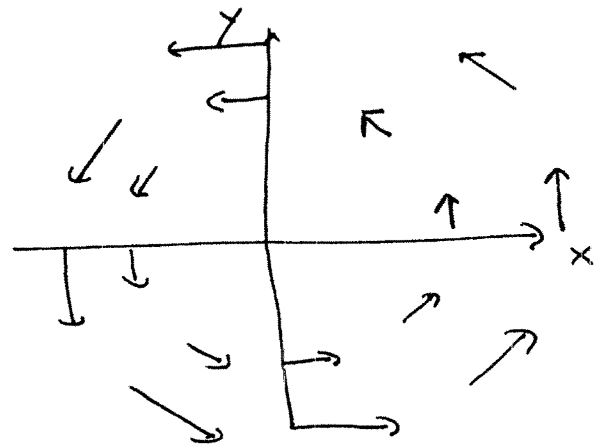
$$\nabla \frac{1}{r^3} = -3 \frac{1}{r^4} \hat{r} \quad \text{(spherical)}$$

$$\nabla V = (\vec{p} \cdot \vec{r}) \left(-3 \frac{1}{r^4} \hat{r} \right) + \frac{1}{r^3} \vec{p}$$

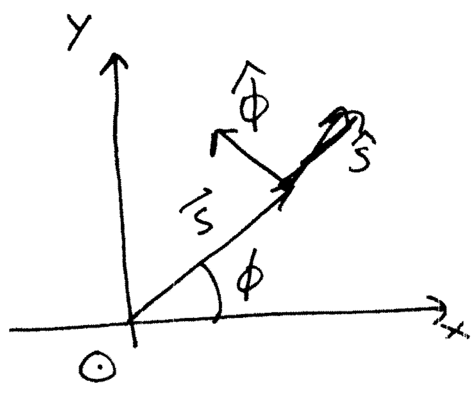
$$= (\vec{p} \cdot \hat{r}) \left(-3 \frac{1}{r^3} \hat{r} \right) + \frac{1}{r^3} \vec{p}$$

$$= \frac{1}{r^3} \left(\vec{p} - 3 (\vec{p} \cdot \hat{r}) \hat{r} \right)$$

E.1.2 $\vec{E} = \gamma\,\hat{z}\times\vec{r}$



In x-y plane $\vec{r} = \vec{s}$ and we can use cylindrical coordinates.



$\hat{s}\times\hat{\phi} = \hat{z}$ $\cancel{\hat{z}\times\hat{\phi} = \hat{s}}$ $\hat{z}\times\hat{s} = \hat{\phi}$

using the coordinate system drawn above.

So $\quad \vec{E} = \gamma\,\hat{z} \times \vec{r} = \gamma\,\hat{z} \times \vec{s}$

$$= \gamma s\,\hat{z} \times \hat{s} = \gamma s\,\hat{\phi}$$

Integrate around circle of radius R

$$\oint_C \vec{E} \cdot d\vec{\ell} = \oint_C (\gamma s\,\hat{\phi}) \cdot (s\,d\phi\,\hat{\phi}) \qquad d\vec{\ell} = s\,d\phi\,\hat{\phi}, \quad s = R$$

$$= \gamma R^2 \oint_C d\phi$$

$$= 2\pi\gamma R^2$$

<u>Stoke's Thm</u> Compute Curl

$$\hat{z} \times \vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ x & y & z \end{vmatrix}$$

$$= \hat{x}(0 - y) - \hat{y}(0 - x) + \hat{z}(0 - 0)$$

$$= -y\,\hat{x} + x\,\hat{y}$$

Compute Curl

$$\nabla \times \vec{E} = \gamma \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix}$$

$$= \gamma \left[\hat{x}(0-0) - \hat{y}(0-0) + \hat{z}(1+1) \right]$$

$$= 2\gamma \hat{z}$$

Integrate over surface of radius R

$$d\vec{a} = \hat{z}\, da = \cancel{\hat{z}\, dr\, (r\, d\phi)}$$

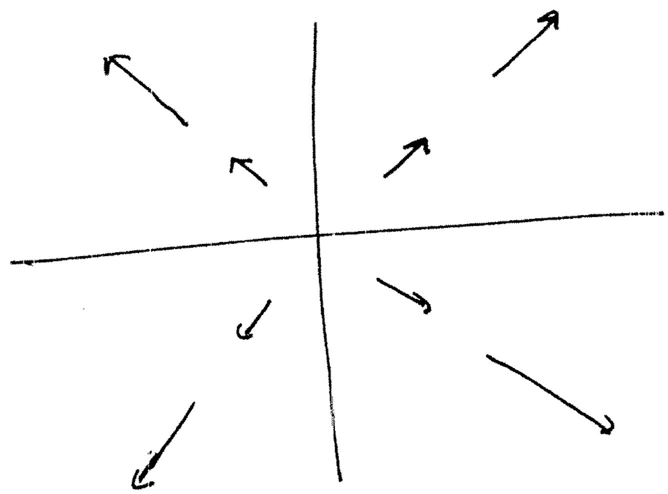
$$= \hat{z}(ds)(d\phi\, s)$$

$$\int_S (\nabla \times \vec{E}) \cdot d\vec{a} = \int_S (2\gamma \hat{z}) \cdot (\hat{z}\, s\, ds\, d\theta)$$

$$= 2\gamma \int_0^{2\pi} d\phi \int_0^R s\, ds$$

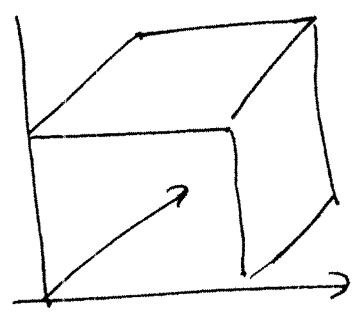
$$= 2\gamma \frac{R^2}{2} \int_0^{2\pi} d\phi = 2\pi \gamma R^2 \quad \checkmark$$

E.1.3 $\quad \vec{E} = \gamma \vec{r}$



Φ out of $0 < x < 1, \quad 0 < y < 1, \quad 0 < z < 1$

cube



$\vec{E} \perp \hat{n}$ for the sides in the coordinate planes.

The other 3 sides are symmetrically placed,

so $\Phi = 3\,\Phi_{top}$

$$\Phi_{top} = \int_{top} \vec{E} \cdot d\vec{a}$$

$$d\vec{a} = \hat{z}\, dx\, dy$$

$$\vec{E} = \gamma(x, y, z)$$

$$\vec{E} \cdot d\vec{a} = \gamma z\, dx\, dy$$

$$\Phi_{top} = \int \vec{E} \cdot d\vec{a} = \int_0^1 dx \int_0^1 dx\ z\gamma \qquad z = 1$$

$$= 1\gamma$$

$$\Phi = 3\,\Phi_{top} = 3\gamma$$

<u>Divergence</u>

$$\nabla \cdot \vec{E} = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \cdot \gamma r\right) + \text{zero stuff}$$

$$= 3\gamma \qquad \vec{E} = \gamma \vec{r} = \gamma r \hat{r}$$

$$\int \nabla \cdot \vec{E} \, d\tau = \int_0^1 dx \int_0^1 dy \int_0^1 dz \, 3\gamma$$

$$= 3\gamma \checkmark$$

E.1.4 $\quad f = x^2 + y^2 + z^2 = r^2$

Cartesian

$$\nabla f = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)(x^2+y^2+z^2)$$

$$= (2x, 2y, 2z)$$

$$= 2\vec{r}$$

Spherical

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} \quad + \text{ zero stuff}$$

$$= 2r\hat{r} = 2\vec{r} \checkmark$$