Homework 11

Due Thursday 5/2/2013 - at beginning of class

Griffiths' 4 Problems (3rd Edition numbers are the same)

7.18

7.26 (Griffiths 3rd Edition 7.24)

7.28 (Griffiths 3rd Edition 7.26)

7.30 (Griffiths 3rd Edition 7.28)

7.36 (Griffiths 3rd Edition 7.33)

8.1 Ex. 7.13 only

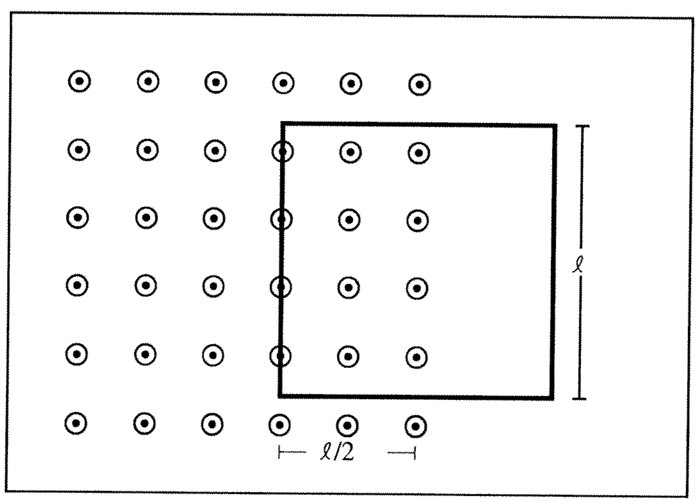
8.4 (a) only

8.7 (Griffiths 3rd Edition 8.5)

Additional Problems

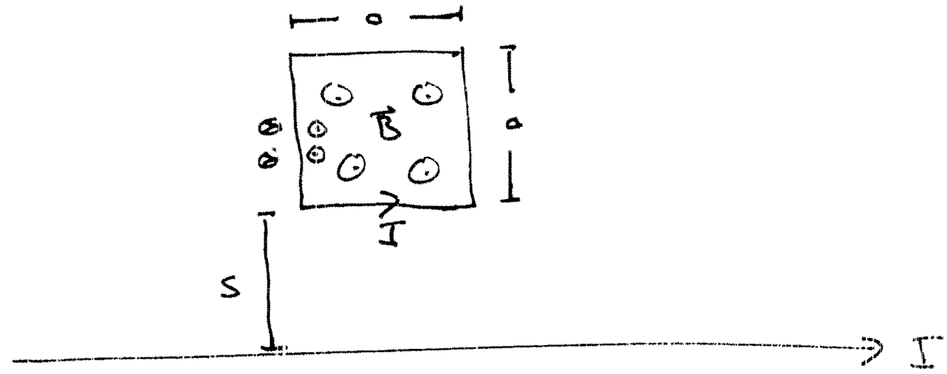
Problem E.11.1 Consider two concentric strongly conducting spherical shells where the smaller has outer radius a and the larger inner radius c. The volume between the shells is filled with two weakly conducting materials. The space from $r = a$ to $r = b$ contains a material with conductivity σ_1. The space from $r = b$ to $r = c$ contains a material with conductivity σ_2. Compute the resistance between the shells (between $r = a$ and $r = c$). Note, $a < b < c$.

Problem E.11.2 An aluminum square with resistivity $2.65 \cdot 10^{-8}\Omega$m is placed halfway inside a magnetic field. The magnetic field is being turned off and the magnitude of the field obeys $B_0 e^{-t/\tau}$ where $\tau = 2$s and $B_0 = 0.2$T. The aluminum square has side length $\ell = 2$cm and cross-sectional area $A = 1cm^2$. Compute the force exerted on the loop at time $t = 0$. Does the force tend to push the loop out of the field or draw the loop into the field?



Problem E.11.3 The region between $z = -a$ and $z = +a$ contains a changing electric field $\vec{E} = E_0 \sin(\omega t)\hat{x}$. Compute the magnetic field at points $z > a$.

7.18



$$I(t) = (1-\alpha t)I \qquad t < \frac{1}{\alpha}$$

The magnetic field through the loop points out of the page. The flux out of the page decreases as the current decreases. The induced current produces a flux that opposes this change in flux, the field of the current is drawn above. By the RHR, the induced current must flow counterclockwise.

The magnetic field through the loop is

$$B = \frac{\mu_0 I}{2\pi s}$$

The flux is

$$\Phi_m = \cancel{BA = \frac{\mu_0 I a^2}{2\pi s}}$$

$$= a\int_s^{s+a} B\, ds$$

$$\Phi_m = a \int_s^{s+a} \frac{\mu_0 I}{2\pi s}\, ds$$

$$= \frac{\mu_0 I a}{2\pi} \ln\left(\frac{s+a}{s}\right)$$

The emf induced in the loop is, by Faraday

$$emf = -\frac{d\Phi_m}{dt} = \frac{\mu_0 a}{2\pi} \ln\left(\frac{s+a}{s}\right) \frac{dI}{dt}$$

If the resistance of The loop is R, the current induced is

$$I_{ind} = \frac{emf}{R} = \frac{\mu_0 a}{2\pi R} \ln\left(\frac{s+a}{s}\right) \frac{dI}{dt}$$

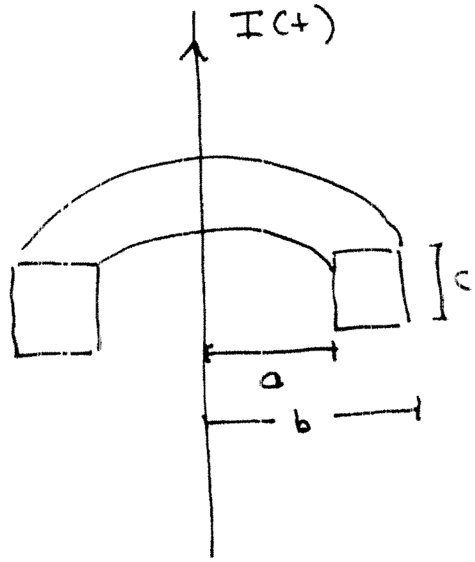
The total charge that flow past some point in the loop is the integral of the current over time

$$|Q| = \left| \int I_{ind}\, dt \right| = \left| \frac{\mu_0 a}{2\pi R} \ln\left(\frac{s+a}{s}\right) \frac{dI}{dt}\, dt \right|$$

$$= \frac{\mu_0 a}{2\pi R} \ln\left(\frac{s+a}{s}\right) \left| \int_{I_0}^{0} dI \right|$$

$$= \frac{\mu_0 a I_0}{2\pi R} \ln\left(\frac{s+a}{s}\right)$$

7.26



$N = 1000$

$a = 1\text{cm}$

$b = 2\text{cm}$

$c = 1\text{cm}$

$I = I_0 \cos \omega t$

$I_0 = \frac{1}{2} A$

$f = 60 Hz$

$\omega = 2\pi f$

$R = 500\,\Omega$

Magnetic field of the wire

$$B_w = \frac{\mu_0 I}{2\pi s}$$

The magnetic flux through the solenoid

$$\Phi_m = N c \int_a^b B \, ds$$

$$= \frac{\mu_0 N I c}{2\pi} \ln\left(\frac{b}{a}\right)$$

The emf induced in the toroid is by Faraday

$$emf = -\frac{d\Phi_m}{dt} = -\frac{\mu_0 N c}{2\pi} \ln\left(\frac{b}{a}\right) \frac{dI}{dt}$$

$$= \frac{I_0 \mu_0 N C}{2\pi} \omega \ln\left(\frac{b}{a}\right) \sin \omega t$$

$$= I_0 \mu_0 N c f \ln\left(\frac{b}{a}\right) \sin \omega t$$

Putting in the stated numbers,

$$\text{emf}_{max} = (0.5A)\left(4\pi\times10^{-7}\frac{Tm}{A}\right)(1000)(0.01m)(60\,s^{-1})\ln(2)$$

$$= 2.6\times10^{-4}V$$

So the current induced is

$$I_{max} = \frac{\text{emf}_{max}}{R} = 5.2\times10^{-7}A$$

$$I(t) = I_{max}\sin\omega t$$

(b) The back current can be calculated from the self-inductance given in example 7.11

$$L = \frac{\mu_0 N^2 c}{2\pi}\ln\left(b/a\right)$$

$$= \frac{\left(4\pi\times10^{-7}\frac{Tm}{A}\right)(1000)^2(0.01)}{2\pi}\ln(2)$$

$$= 1.386\times10^{-3}H$$

Back emf

$$\mathcal{E}_{back} = -L\frac{dI_{ind}}{dt} = \underbrace{-L I_{max}}_{\mathcal{E}_{back,\,max}}\omega\cos\omega t$$

$$emf_{back,\,max} = -L\,I_{max}\,\omega = -2\pi f L\,I_{max}$$

$$= -2\pi\,(60\,s^{-1})(5.2\times10^{-7}A)(1.386\times10^{-3}H)$$

$$= 2.7\times10^{-7}\,V$$

The back current

$$I_{back,\,max} = \frac{emf_{back,\,max}}{R} = 5.43\times10^{-10}A$$

Ratio of back emf to direct emf

$$= \frac{2.7\times10^{-7}V}{2.6\times10^{-4}V} \sim \frac{1}{1000}$$

7.28

(a) $L = \mu_0 n^2 A l$

$$U = \frac{1}{2} L I^2 = \frac{1}{2}\mu_0 n^2 A l I^2 = \frac{1}{2}\mu_0 n^2 \pi R^2 l I^2$$

(b) $W = \frac{1}{2}\oint \vec{A}\cdot\vec{I}\, dl$

$$\vec{A} = \frac{\mu_0 n I}{2} R\,\hat{\phi}$$

The integral is carried out around the circumference of one ring of thickness dz and gives the energy in one ring

$$d\vec{I} = K\, dz\,\hat{\phi} = n I\, dz\,\hat{\phi}$$

$$dW = \frac{1}{2}\oint (\vec{A}\cdot d\vec{I})\, dl$$

$$W = \int_0^l dW = \frac{1}{2}\underbrace{\int_0^l dz}_{l}\ \underbrace{\oint_c \vec{A}\cdot(nI\hat{\phi})\, dl}_{2\pi R A n I}$$

$$= \frac{1}{2} l \cdot 2\pi R \cdot \frac{\mu_0 n I R}{2} n I$$

$$= \frac{1}{2} n^2 I^2 \mu_0 \pi R^2 l$$

(c)

$$W = \frac{1}{2\mu_0} \int B^2 d\tau = \frac{1}{2\mu_0} B^2 \cdot \pi R^2 l$$

$$= \frac{1}{2\mu_0} \cdot (\mu_0 n I)^2 \cdot \pi R^2 l = \frac{1}{2} \mu_0 n^2 I^2 \pi R^2 l$$

(d)

$$W = \frac{1}{2\mu_0} \left[\int_V B^2 d\tau - \oint_S (\vec{A} \times \vec{B}) \cdot d\vec{\sigma} \right.$$

where the volume V is $a < s < b$

where $a < R$ and $b > R$.

The first term is as before

$$\frac{1}{2\mu_0} \int B^2 d\tau = \frac{1}{2\mu_0} (\mu_0 n I)^2 \cdot \pi (R^2 - a^2) l$$

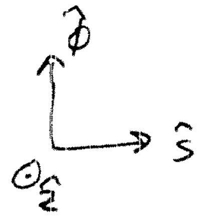
where $\pi (R^2 - a^2) l$ is the volume of the region

where the field is non-zero.

The second term

$$\vec{A} \times \vec{B} = \frac{\mu_0 n I}{2} s \hat{\phi} \times \mu_0 n I \hat{z} \qquad s < R$$

$$= 0 \qquad s > R.$$



$$\hat{\phi} \times \hat{z} = \hat{s}$$

$$\vec{A} \times \vec{B} = \frac{1}{2} \mu_0^2 n^2 I^2 s \; \hat{s} \qquad s < R$$

$\int_S (\vec{A} \times \vec{B}) \cdot d\vec{a}$ must be taken over the inner and outer surface. The integral over the outer surface is zero. For the inner surface, $d\vec{a} = -\hat{s}\, da$

$$\frac{1}{2\mu_0} \int_S \vec{A} \times \vec{B} \cdot (-\hat{s}\, da) = \cancel{\frac{1}{2\mu_0} \cdot \frac{1}{2} \mu_0^2 n^2 I^2 \int_0^{2\pi} d\phi \, R\ell}$$

$$= -\frac{1}{2\mu_0} \cdot \frac{1}{2} \mu_0^2 n^2 I^2 a \underbrace{\int_S da}_{2\pi a \ell}$$

$$= -\frac{1}{2\mu_0} \cdot \frac{1}{2} \mu_0^2 n^2 I^2 a \cdot 2\pi a \ell$$

$$= -\frac{1}{2} \mu_0 n^2 I^2 \pi a^2 \ell$$

$$W = \frac{1}{2\mu_0} \int_V B^2 d\tau - \frac{1}{2\mu_0} \int_S \vec{A} \times \vec{B} \cdot d\vec{a}$$

$$= \frac{1}{2} \mu_0 n^2 I^2 \pi R^2 \ell$$

7.30 There is no obvious correct surface through which the flux should be calculated. As such, it is unclear how to compute L from Φ_m / I.

We can however unambiguously compute the energy U and find L from $U = \frac{1}{2} L I^2$.

The magnetic field in the wire with $J = I / \pi R^2$

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = 2\pi s B = \mu_0 I_{enc} = \mu_0 \pi s^2 J = \mu_0 \frac{s^2}{R^2}$$

$$B = \frac{\mu_0 I s}{2\pi R^2} \quad \text{as before}$$

The energy in a length l of the wire is

$$U = \frac{1}{2\mu_0} \int_V B^2 \, d\tau = \frac{l}{2\mu_0} \int B^2 \, da$$

$$da = ds \, s \, d\phi$$

$$= \frac{1}{2\mu_0} \int \left(\frac{\mu_0 I s}{2\pi R^2} \right)^2 \cdot s \, ds \, d\phi$$

$$= \frac{1}{2\mu_0} \left(\frac{\mu_0 I}{2\pi R^2} \right)^2 \int_0^{2\pi} d\phi \int_0^R s^3 \, ds$$

$$U = \frac{\ell}{2\mu_0}\left(\frac{\mu_0 I}{2\pi R^2}\right)^2 \cdot 2\pi \cdot \frac{R^4}{4}$$

$$= \frac{1}{2}\left(\frac{\mu_0 \ell}{8\pi}\right) I^2 \quad = \quad \frac{1}{2} L I^2$$

$$L = \frac{\mu_0 \ell}{8\pi}$$

7.36

$$\vec{J}_d = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$= \frac{\epsilon_0 \mu_0 I_0 \omega}{2\pi} \ln\left(\frac{a}{s}\right) \hat{z} \frac{\partial}{\partial t} \sin\omega t$$

$$= + \frac{\epsilon_0 \mu_0 I_0 \omega^2}{2\pi} \ln\left(\frac{a}{s}\right) \cos\omega t \; \hat{z}$$

(b) Total displacement current

$$I_d = \int J_d \cdot d\vec{a} \qquad da = s\, ds\, d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^a s\, J_d\, ds$$

$$= 2\pi \cdot \left(\frac{+\epsilon_0 \mu_0 I_0 \omega^2}{2\pi} \cos\omega t \right) \underbrace{\int_0^a s \ln\left(\frac{a}{s}\right) ds}_{+a^2/4}$$

$$= \epsilon_0 \mu_0 \omega^2 I_0 \cos\omega t \cdot \left(+ \frac{a^2}{4} \right)$$

$$= \frac{1}{4} \epsilon_0 \mu_0 a^2 \omega^2 I_0 \cos\omega t$$

(c) $I = I_0 \cos \omega t$

$$\frac{I_d}{I} = \frac{1}{4} \varepsilon_0 \mu_0 a^2 \omega^2$$

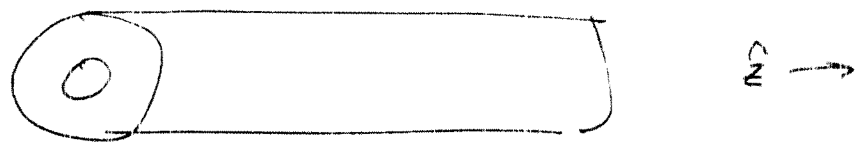
If $$\frac{I_d}{I} = 0.01 = \frac{1}{4} \mu_0 \varepsilon_0 a^2 (2\pi f)^2$$

$$= \mu_0 \varepsilon_0 a^2 \pi^2 f^2 \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$f = \left(\frac{0.01}{\mu_0 \varepsilon_0 a^2 \pi^2} \right)^{1/2} = \frac{1}{10} \frac{c}{a \pi}$$

$$= \frac{3 \times 10^8 \text{ m/s}}{10 \cdot (0.002\text{m}) \cdot \pi} = 1.5 \times 10^9 \text{ Hz}$$

(8.1)



Magnetic field – $\dfrac{\mu_0 I}{2\pi s}\,\hat{\phi}$ Ex 7.13

Electric field – If a potential difference ΔV is established between the inner and outer conductor, a charge density λ is established on inner conductor. The electric field between the conductors is then

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 s}\,\hat{s}$$

~~The po.~~ The potential difference between the conductors is then

$$\Delta V = -\int_a^b \vec{E}\cdot d\vec{l} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

The Poynting vector is

$$|\vec{S}| = \frac{1}{\mu_0}\left|\vec{E}\times\vec{B}\right| = \frac{EB}{\mu_0}$$

since $\vec{E}\perp B$

The energy flow down the channel per unit time, the power is

$$P = \int S \, da = 2\pi \int_a^b s \, S \, ds$$

$$S = \frac{1}{\mu_0} E B = \left(\frac{1}{\mu_0}\right)\left(\frac{\lambda}{2\pi\epsilon_0 s}\right)\left(\frac{\mu_0 I}{2\pi s}\right)$$

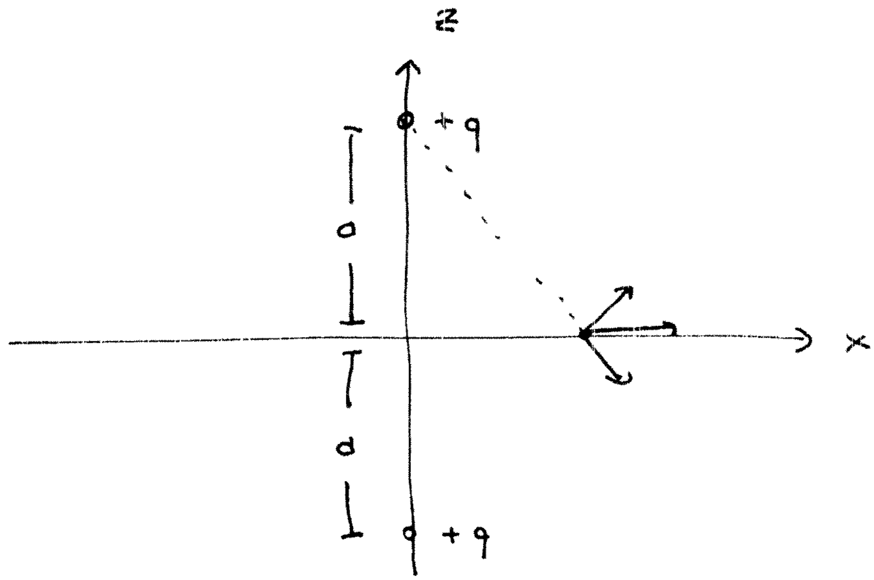
$$= \frac{\lambda I}{4\pi^2 \epsilon_0 s^2}$$

$$P = 2\pi \int_a^b \frac{\lambda I}{4\pi^2 \epsilon_0 s^2} \cdot s \, ds$$

$$= \frac{\lambda I}{2\pi\epsilon_0} \int_a^b \frac{ds}{s} = \frac{\lambda I}{2\pi\epsilon_0} \ln\left(b/a\right)$$

$$= I \Delta V \quad \checkmark$$

8.4 (a)



The electric field of the two charges on the plane is

$$\vec{E} = \frac{2kq}{s^2+a^2} \cdot \frac{s}{\sqrt{s^2+a^2}} \hat{s}$$

$$= \frac{2kq\,s}{(s^2+a^2)^{3/2}} \hat{s}$$

$$\vec{F} = \int \overleftrightarrow{T} \cdot d\vec{a}$$

$$d\vec{a} = \hat{z}\, da$$

$$\overleftrightarrow{T} \cdot d\vec{a} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ da \end{pmatrix}$$

$$\vec{F} = \begin{pmatrix} T_{xz}\, da \\ T_{yz}\, da \\ T_{zz}\, da \end{pmatrix}$$

The force must be in the positive z direction,

so $\int T_{xz}\, da = \int T_{yz}\, da = 0$

$$T_{zz} = \epsilon_0 \left(E_z^2 - \frac{1}{2} E^2 \right) + \frac{1}{\mu_0} \left(B_z^2 - 1/2\, B^2 \right)$$

$$= \epsilon_0 \left(E_z^2 - \frac{1}{2} \vec{E} \cdot \vec{E} \right)$$

$$E_z = 0 \quad , \quad \vec{E} \cdot \vec{E} = \frac{4 k^2 q^2 s^2}{(s^2 + a^2)^3} = \frac{q^2 s^2}{4 \pi^2 \epsilon_0^2 (s^2 + a^2)^3}$$

$$T_{zz} = -\frac{1}{2} \epsilon_0 \vec{E} \cdot \vec{E} = \frac{-q^2 s^2}{8 \pi^2 \epsilon_0 (s^2 + a^2)^3}$$

To calculate the force on the upper charge we need the outward normal from the top plane

$$\hat{n} = -\hat{z}$$

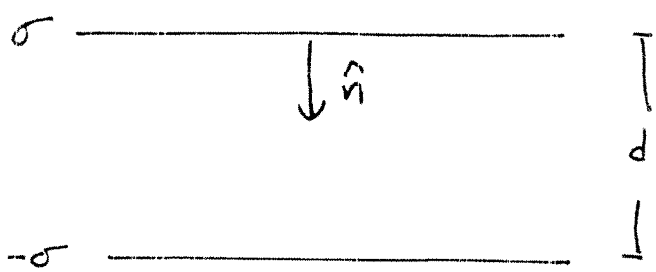
$$\vec{F} = (-\hat{z})\int T_{zz}\, da$$

$$= \frac{q^2\hat{z}}{8\pi^2\varepsilon_0}\int_{\text{plane}} \frac{s^2}{(s^2+a^2)^3}\, ds\, s\, d\phi$$

$$= \frac{2\pi q^2\hat{z}}{8\pi^2\varepsilon_0}\underbrace{\int_0^\infty \frac{s^3}{(s^2+a^2)^3}}_{\frac{1}{4a^2}}$$

$$\vec{F} = \frac{q^2\hat{z}}{16\pi a^2\varepsilon_0} = \frac{q^2}{4\pi\varepsilon_0(z_0)^2}\hat{z} \quad \checkmark$$

8.7



$$\vec{E} = -\frac{\sigma}{\varepsilon_0}\,\hat{z}$$

(a) $$T_{ij} = \varepsilon_0\left(E_i E_j - \frac{1}{2}\delta_{ij} E^2\right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2\right)$$

$$E_{zz} = -\frac{\sigma}{\varepsilon_0}$$ All other $E_{ij}, B_{ij} = 0$

$$T_{ij} = 0 \quad \text{if} \quad i \neq j$$

$$T_{xx} = \varepsilon_0\left(-\frac{1}{2}E^2\right) = -\frac{\sigma^2}{2\varepsilon_0} = T_{yy}$$

$$T_{zz} = \varepsilon_0\left(E_z^2 - \frac{1}{2}E^2\right) = \frac{\sigma}{2\varepsilon_0}$$

$$\overleftrightarrow{T} = \frac{\sigma}{2\varepsilon_0}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b) The force on the top plate

$$\vec{F} = \int \overleftrightarrow{T} \cdot d\vec{a} \qquad d\vec{a} = -\hat{z}\,dx\,dy$$

Force per unit area, $\vec{f} = \dfrac{\vec{F}}{da} = \overleftrightarrow{T} \cdot (-\hat{z})$

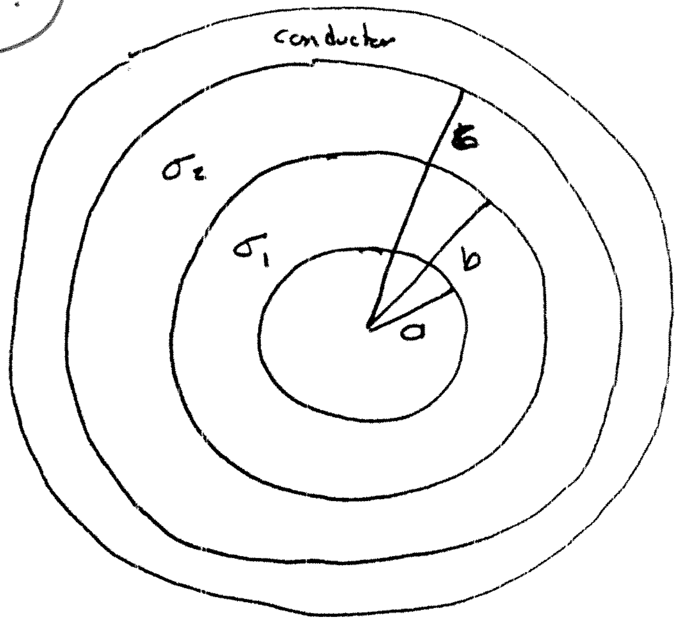
$$= -\hat{z}\left(\frac{\sigma^2}{2\epsilon_0}\right)$$

$$\vec{f} = \frac{\sigma^2}{2\epsilon_0}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \frac{\sigma^2}{2\epsilon_0}\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

(c) The momentum per unit area per unit time crossing x-y plane is $-T_{zz} = -\dfrac{\sigma^2}{2\epsilon_0}$

(d) The recoil force $\vec{f}$ is $\dfrac{dp}{dt} = -T_{zz} = \dfrac{-\sigma^2}{2\epsilon_0}\,\hat{z}$

E.11.1



Assume I flows through all cross sections. The current density is

$$\vec{J} = \frac{I}{4\pi r^2}\hat{r}$$

By Ohm's Law, the conductivity is related to the current by

$$\vec{J}_1 = \sigma_1 \vec{E}$$

$$\vec{E}_1 = \frac{\vec{J}}{\sigma_1} = \frac{I}{4\pi r^2 \sigma_1}\hat{r}$$

The potential difference between the shell and radius b is then

$$\Delta V_1 = -\int \vec{E}_1 \cdot d\vec{l} = -\frac{I}{4\pi\sigma_1}\int_a^b \frac{dr}{r^2}$$

$$\Delta V_1 = \frac{I}{4\pi\sigma_1}\left(\frac{1}{b} - \frac{1}{d}\right)$$

which is correctly negative.

~~The resistance~~ Likewise, the potential difference across the second conductor is

$$\Delta V_2 = \frac{I}{4\pi\sigma_2}\left(\frac{1}{c} - \frac{1}{b}\right)$$

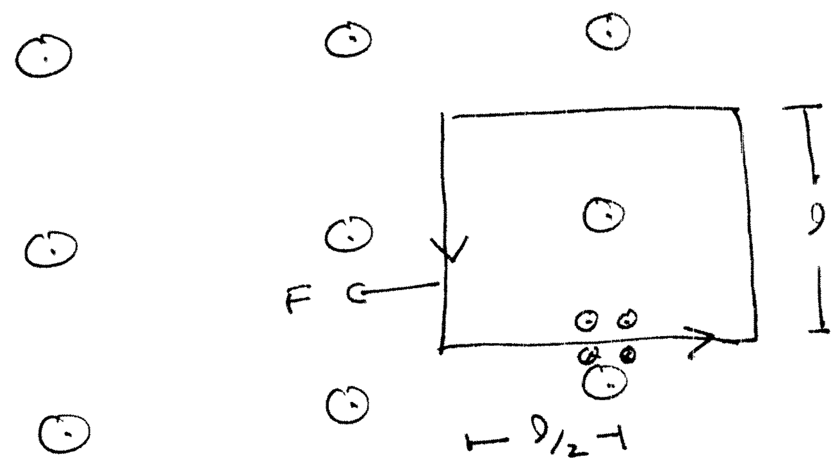
The resistance is then

$$R = \frac{|\Delta V_1 + \Delta V_2|}{I} = \frac{1}{4\pi}\left(\frac{1}{\sigma_1}\left(\frac{1}{d} - \frac{1}{b}\right) + \frac{1}{\sigma_2}\left(\frac{1}{b} - \frac{1}{c}\right)\right)$$

Dimensions are correct

$$[R] = \left[\frac{l}{\sigma A}\right] = \left[\frac{1}{\sigma}\right]\left[\frac{1}{m}\right]$$

E.11.2



Field decreasing, to oppose decrease the induced current must produce field to oppose change (Lenz)
So the induced current is CCW.

The force on the top and bottom cancel, so the only force is that on the left side, $\vec{F} = I\vec{L} \times \vec{B}$. By RHR, the force is to the left of the page and the loop is drawn into the field.

The resistance of the loop is $R = \dfrac{\rho(4D)}{A}$

$$R = \frac{(2.65\times10^{-8}\,\Omega\text{m})(4)(0.02\text{m})}{1\times10^{-4}\text{m}^2}$$

$$= 2.12\times10^{-5}\,\Omega$$

The emf is given by Faraday's Law

$$\text{emf} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} B\frac{l^2}{2}$$

$$= -\frac{l^2 B_0}{2}\frac{d}{dt} e^{-t/\tau} = \frac{l^2 B_0}{2\tau} e^{-t/\tau}$$

At $t=0$,

$$\text{emf} = \frac{l^2 B_0}{2\tau} = \frac{(0.02\text{m})^2(0.2\text{T})}{2(2\text{s})} = 2\times10^{-5}\text{V}$$

The current in the loop is then

$$I = \frac{\text{emf}}{R} = \frac{2\times10^{-5}\text{V}}{2.12\times10^{-5}\Omega} = 0.943\text{A}$$

The force is given by the Lorentz force

$$|\vec{F}| = |I\vec{L}\times\vec{B}| = IlB = (0.943\text{A})(0.02\text{m})(0.2\text{T})$$

$$= 0.0038\text{N}$$

<u>Symbolically</u>

$$F(0) = IlB = \left(\frac{l^2 B_0}{2\tau R}\right)B_0 l = \frac{l^3 B_0^2}{2\tau(\rho 4l/A)}$$

$$= \frac{l^2 B_0^2 A}{8\tau\rho}$$

The emf is given by Faraday's Law

$$\text{emf} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\left(\frac{l^2}{2}\right)B$$

$$= -\frac{l^2 B_0}{2}\frac{d}{dt}e^{-t/\tau} = \frac{l^2 B_0}{2\tau}e^{-t/\tau}$$

The emf at $t=0$ is then

$$\text{emf} = \frac{l^2 B_0}{2\tau} = \frac{(0.02m)(0.2T)}{2(2s)} = 1\times10^{-3}V$$

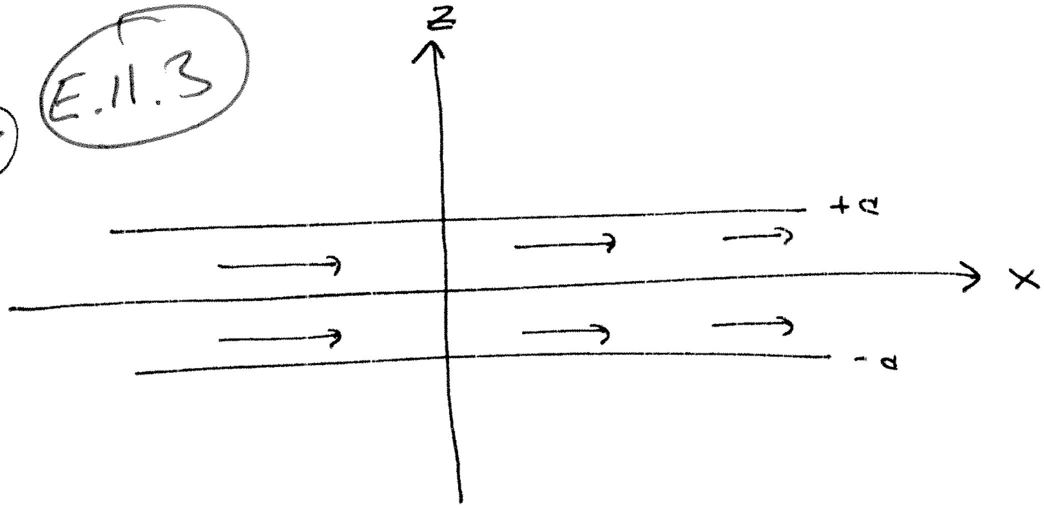
The current in the loop is then

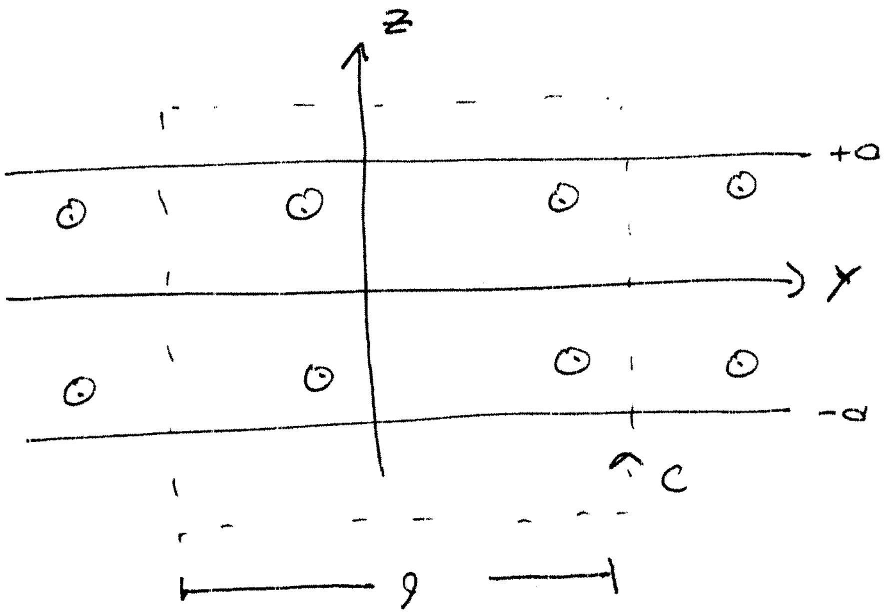
$$I = \frac{\text{emf}}{R} = \frac{1\times10^{-3}V}{2.12\times10^{-5}\Omega} = 47.2A$$

The force is given by the Lorentz force

$$F = ILB = (47.2A)(0.02m)(0.2T) = 0.19N$$

~~4~~ E.11.3





The electric flux through the surface bounded by C

is $\Phi_e = EA = E \cdot 2a \cdot l$

$$= 2alE_0 \sin\omega t$$

The displacement current is

$$I_d = \varepsilon_0 \frac{d\Phi_e}{dt} = 2\varepsilon_0 \omega a l E_0 \cos\omega t$$

Using the RHR, for a displacement current out of the page in the second figure, the magnetic field above $z > a$, points to the left

$$\vec{B}_t = -B_0 \hat{y}$$

and the field below

$$\vec{B}_b = B_0 \hat{y}$$

<u>Amperes Law</u>

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 (I_{enc} + I_d) = \mu_0 I_d$$

$$= + B_0 l + B_0 l = \mu_0 I_d$$

$$B_0 = \frac{\mu_0 I_d}{2l} = \mu_0 \varepsilon_0 \omega a E_0 \cos \omega t$$

Direction $-\hat{y}$ by RHR wire.