Image Solutions for Dielectrics

Image solutions for a planar dielectric can be developed in analogy to the image solutions for conductors.

Consider the field above an infinite dielectric half space ($z<0$) where a point charge Q is a distance D above the plane.

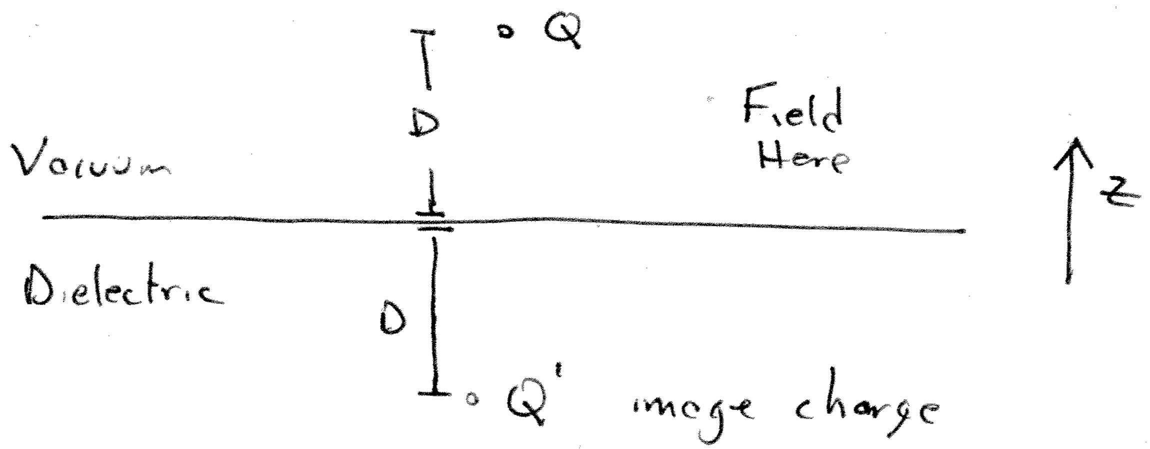


Image charge $Q' = -\frac{\kappa-1}{\kappa+1} Q$ a distance D below the plane $\kappa = \varepsilon_r = 1+\chi_e$

Consider the field within an infinite dielectric half space ($z<0$) with a point charge Q a distance D above the plane.

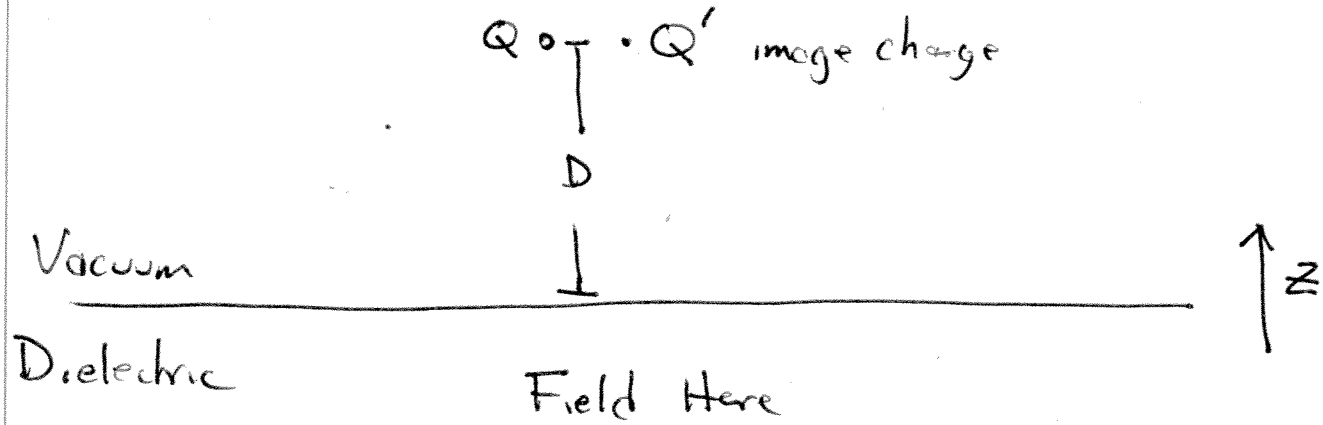
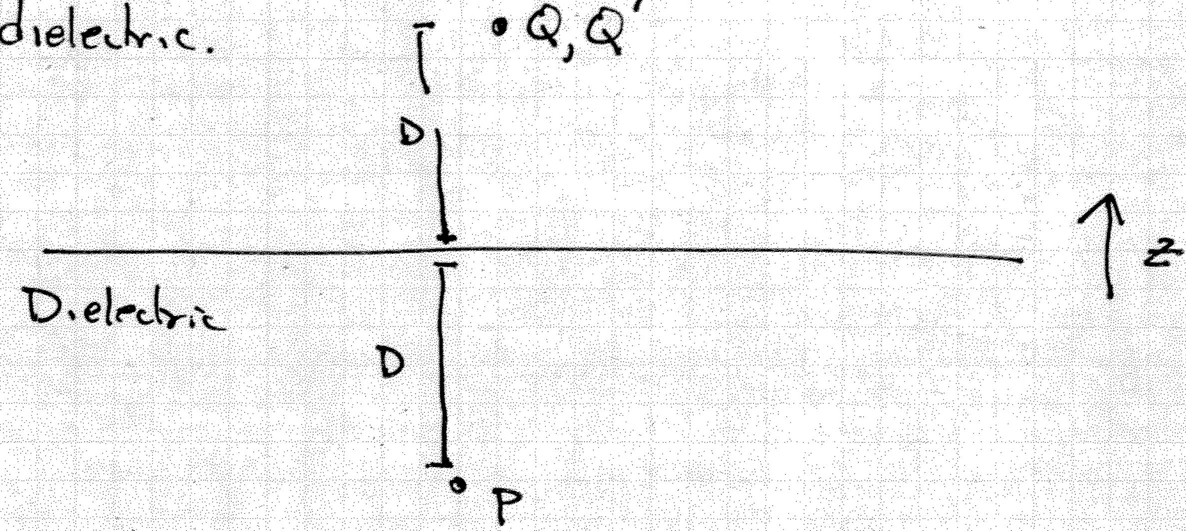


Image Charge Q' at same location as Q

$$Q' = -\left(\frac{K-1}{K+1}\right) Q$$

Ex A point charge Q is a distance D above a planar dielectric with $\varepsilon_r = k = 2$. Compute field a distance D inside the dielectric.



$$Q' = -\left(\frac{k-1}{k+1}\right) Q = -\left(\frac{2-1}{2+1}\right) Q$$

$$= -\frac{1}{3} Q$$

$$\vec{E}_P = \frac{K(Q+Q')}{(2D)^2}(-\hat{z}) = -\frac{kQ}{(2D)^2} \cdot \frac{2}{3}\hat{z}$$

$$\Rightarrow \qquad = \frac{kQ}{6D^2}\hat{z}$$

Dielectric reduces field by $\frac{2}{3}$ not $\frac{1}{2}$.

Polarization within Dielectric

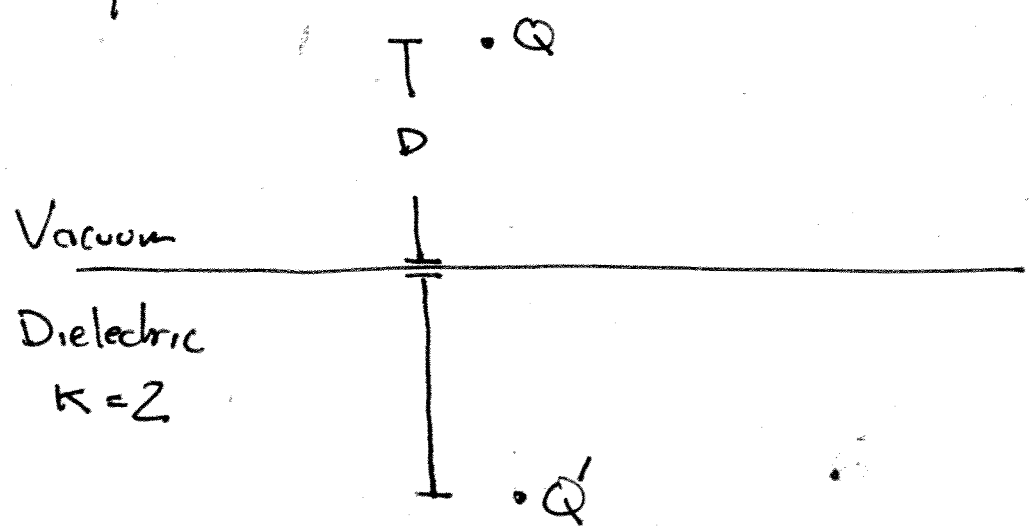
$$\vec{P} = \varepsilon_0 \chi_e \vec{E} = \varepsilon_0 (\kappa - 1)\vec{E}$$

$$= -\varepsilon_0 \cdot \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{Q}{6D^2}\hat{z} \cdot (\kappa - 1)$$

$$= -\frac{Q(\kappa-1)}{24\pi D^2}\hat{z} \qquad \kappa - 1 = 1$$

$$= \frac{-Q}{24\pi D^2}\hat{z} = \frac{-Q}{24\pi D^2}\hat{z}$$

<u>Ex</u> Force on point charge Q due to dielectric plane.



$$Q' = -\left(\frac{\kappa-1}{\kappa+1}\right)Q$$

$$= -\frac{1}{3}Q$$

Force

$$\vec{F}_Q = \frac{1}{4\pi\varepsilon_0} \frac{Q Q'}{(2D)^2} \hat{z}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q\left(-\frac{1}{3}Q\right)}{4D^2} \hat{z}$$

$$= \frac{-Q^2}{48\pi\varepsilon_0 D^2} \hat{z}$$