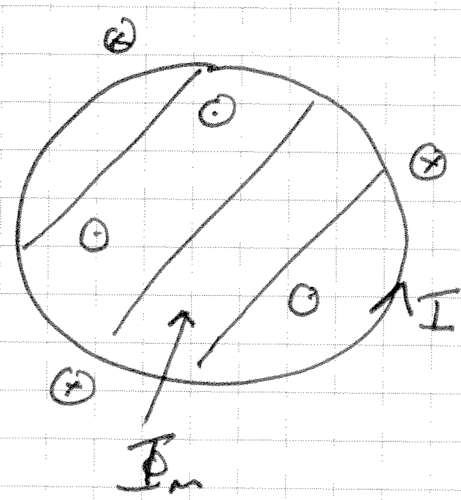
Inductance

The magnetic field is proportional to the current, therefore the flux through a circuit is proportional to the ~~field that p~~ current that produced the field.

<u>Self-Inductance (L)</u> The ratio of the flux through a circuit to the current in the same circuit.

$$L = \frac{\Phi_m}{I}$$

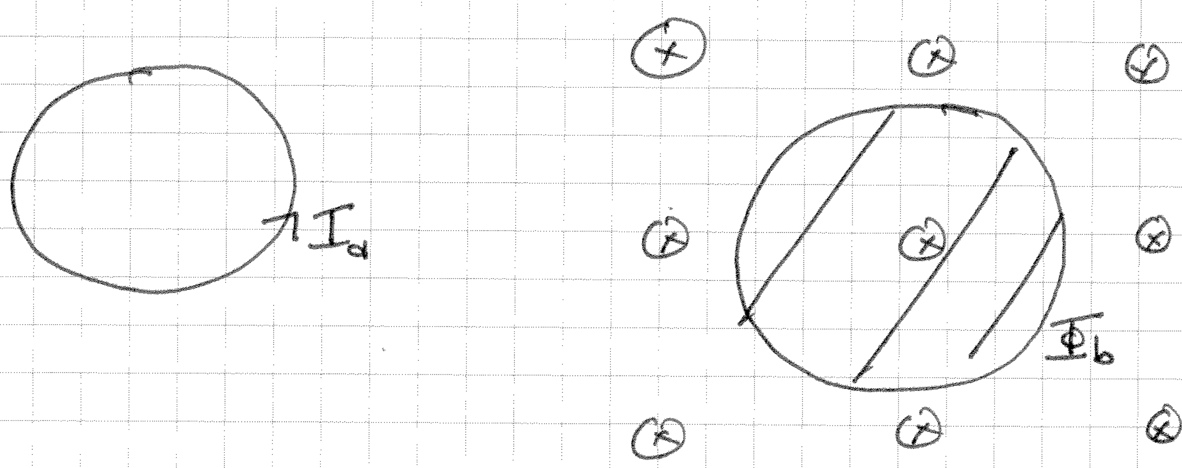


Mutual Inductance (M_{ab}) - The ratio of the flux through circuit b to the current in circuit a that produced the flux.

$$M_{ab} = \frac{\Phi_b}{I_a} = M_{ba}$$

$\Rightarrow$ Both L, M_{ab} are independant of current and depend only on constants and geometry.

$\Rightarrow$ Units Henry $\quad 1\,H = 1\,\frac{Tm^2}{A}$



Field from I_a

Work on M_{ab}

$$\Phi_b = \oint_{C_b} \vec{A}_a \cdot d\vec{l}_b \qquad \vec{A}_a \text{ Vector potential}$$

$$\vec{A}_a = \frac{\mu_0 I_a}{4\pi} \oint_{C_a} \frac{d\vec{l}_a}{r''}$$

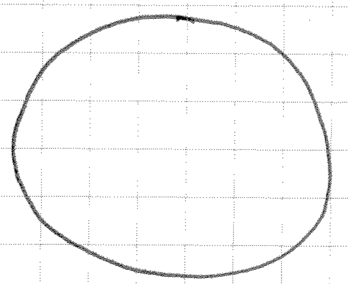
$$\Rightarrow \quad \Phi_b = \frac{\mu_0 I_a}{4\pi} \oint_{C_b} \oint_{C_a} \frac{d\vec{l}_a \cdot d\vec{l}_b}{r''}$$

$$\Rightarrow \quad M_{ab} = \frac{\Phi_b}{I_a} = \underbrace{\frac{\cancel{I}\mu_0}{4\pi} \oint_{C_b} \oint_{C_a} \frac{d\vec{l}_a \cdot d\vec{l}_b}{r''}}_{\text{Geometry + Constants}}$$

$$= M_{ba}$$

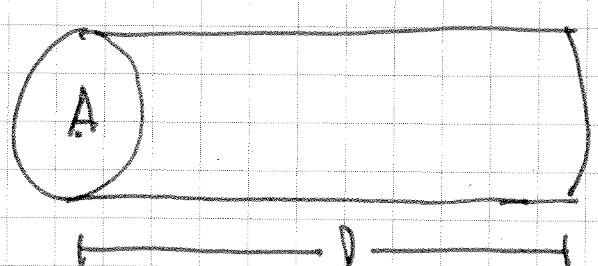
Self-Inductance

Ring



- Too hard

Solenoid — Assume fields are the same as that of an infinite solenoid.



N turns

Let a current I flow in the solenoid

$$B = \mu_0 \frac{N}{l} I = \mu_0 n I$$

$$n = \frac{N}{l}$$

Flux

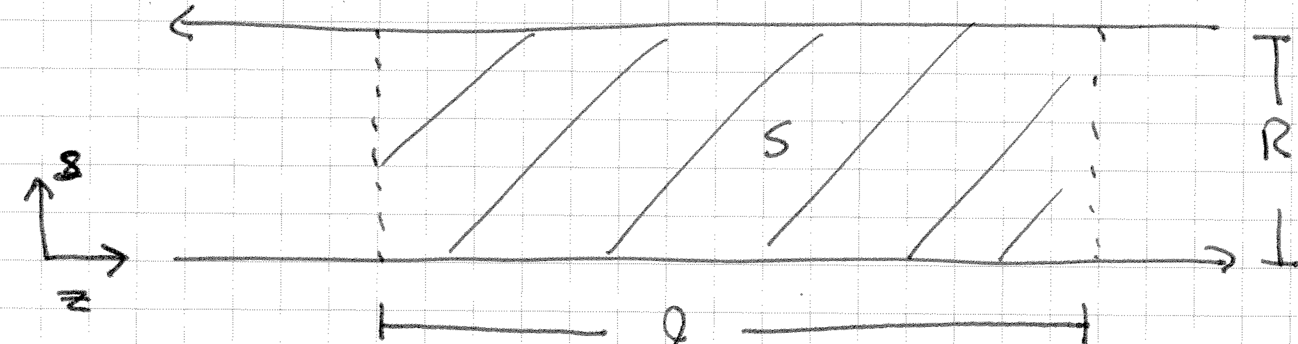
$$\Phi_M = NBA = \mu_0 N n I A$$

$$= \mu_0 n^2 I A l$$

Inductance of Solenoid in Infinite Solenoid Approximation

$$L = \frac{\Phi_m}{I} = \mu_0 n^2 A l$$

Ex Self-Inductance per Unit Length of Long Parallel wires of radius a and separation R.



Introduce length l and form dashed "loop".

Let current I flow in the circuit.

The field between the wires points out of the page and has magnitude

$$B = \frac{\mu_0 I}{2\pi s} + \frac{\mu_0 I}{2\pi (R-s)}$$

where s is the distance from the bottom wire.

The flux through our surface S is

$$\Phi_m = \ell \int_a^{R-a} B\, ds$$

Each wire makes the same contribution to the so this would be twice the flux of one wire

$$\Phi_m = 2\ell \int_a^{R-a} B_1\, ds$$

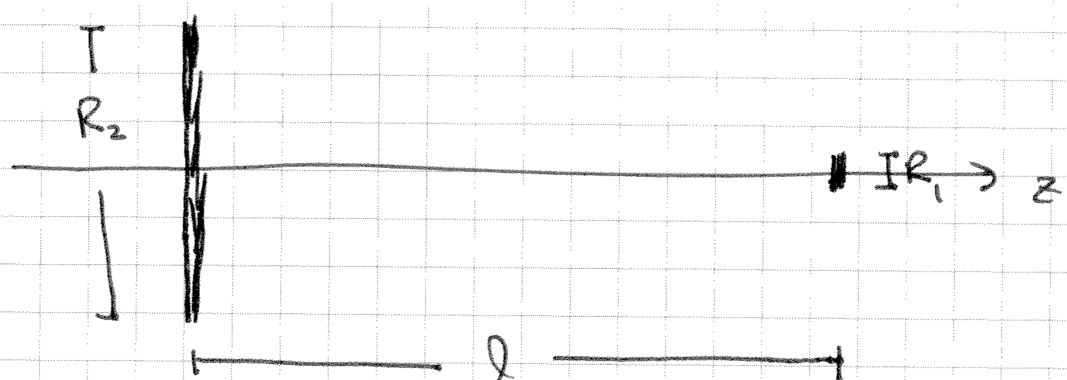
$$= 2\ell \int_a^{R-a} \frac{\mu_0 I}{2\pi s}\, ds$$

$$= \frac{\ell \mu_0 I}{\pi} \ln\left(\frac{R-a}{a}\right)$$

Inductance

$$L = \frac{\Phi_m}{I} = \frac{\mu_0 \ell}{\pi} \ln\left(\frac{R-a}{a}\right)$$

<u>Ex</u> Compute mutual inductance between two co-axial rings of radius R_1, R_2 s.t. $R_1 \ll R_2$. The center of the rings are a distance l apart.



Compute either M_{12} or M_{21}.

Compute $M_{12} = \dfrac{\Phi_1}{I_2}$

Since R_1 is small, we can approximate the field by the field at its center, B_2, due to ring 2.

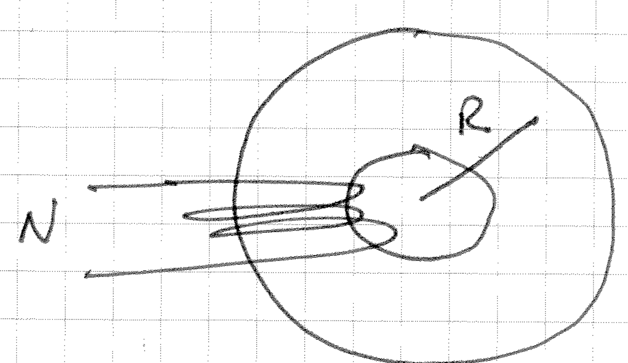
$$\Phi_1 = \pi R_1^2 B_2$$

$$B_2 = \frac{\mu_0 I_2 R_2^2}{2(l^2 + R_2^2)^{3/2}}$$

$$M_{12} = \frac{\Phi_1}{I_2} = \frac{\mu_0 \pi R_1^2 R_2^2}{2(\ell^2 + R_1^2)^{3/2}}$$

$\Rightarrow$ If we calculated M_{21} we would have to integrate the field over the surface bounded by the loop.

Ex Self-Inductance iron ring of cross-section A with N turns of wire.



Compute field if I flows in wire

$$\oint \vec{H} \cdot d\vec{\ell} = NI = 2\pi R H$$

$$H = \frac{NI}{2\pi R}$$

$$B = \mu_0 \mu_r H = \frac{\mu_0 \mu_r N I}{2\pi R}$$

Flux

$$\Phi_m = NBA$$

$$= NA\left(\frac{\mu_0 \mu_r N I}{2\pi R}\right)$$

$$= \frac{\mu_0 \mu_r N^2 A I}{2\pi R}$$

~~Mutual~~ Self Inductance

$$L = \frac{\Phi_m}{I} = \frac{\mu_0 \mu_r N^2 A}{2\pi R}$$