Linear Magnetic Materials

Some materials like copper and aluminum has a linear response to an applied field.

$$\vec{M} = \chi_m \vec{H}$$

__Magnetic Susceptibility__ (χ_m) - Characterizes material response to magnetic fields

Relative Permeability $\mu_r = 1 + \chi_m$

Permeability $\mu = \mu_r \mu_0$

$$\vec{B} = \mu_0 \vec{M} + \mu_0 \vec{H} = \mu_0 \chi_m \vec{H} + \mu_0 \vec{H}$$

$$= \mu_0 (1 + \chi_m) \vec{H}$$

$$= \mu_0 \mu_r \vec{H}$$

$$= \mu \vec{H}$$

Paramagnetic $(\chi_m > 0)$ - Induced moment in same direction as applied field.

Diamagnetic $(\chi_m < 0)$ - Induced moment in opposite direction to applied field.

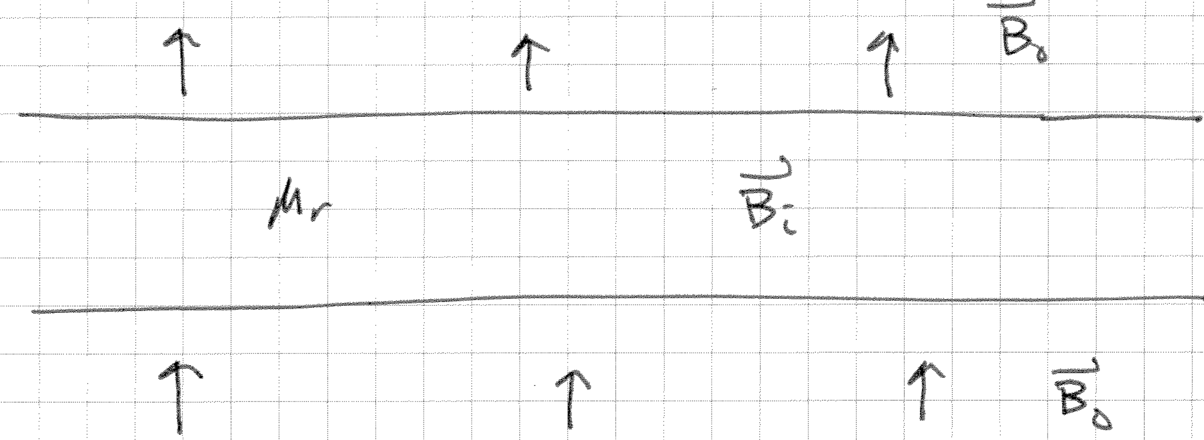
Ex

Copper	$\chi_m = -1 \times 10^{-5}$	Diamagnetic
Polyethylene	$\chi_m = -0.2 \times 10^{-5}$	Diamagnetic
Aluminum	$\chi_m = 2.1 \times 10^{-5}$	Paramagnetic

$\Rightarrow$ Other materials have large non-linear response such as iron $\chi_m \sim 1000$

or superconductor $\chi_m = -1$

Ex Thin slab of magnetic material in uniform applied field $\vec{B}_0$.



$$\nabla \cdot \vec{B} = 0 \quad \Rightarrow \quad B^{\perp}_{outside} = B^{\perp}_{inside}$$

$$\Rightarrow \quad \vec{B}_0 = \vec{B}_i$$

<u>Outside</u> $\quad \vec{M}_0 = 0$

$$\mu_0 \vec{H}_0 = \vec{B}_0$$

<u>Inside</u>

$$\vec{B}_i = \mu_0 \vec{H}_i + \mu_0 \vec{M}_i = \mu_0 \mu_r \vec{H}_i$$

$$= \vec{B}_0 = \mu_0 \vec{H}_0$$

$$\vec{H}_0 = \frac{\vec{H}_i}{\mu_r}$$

$\Rightarrow$ Slab reduces $\vec{H}$ by a factor of μ_r but does not change $\vec{B}$

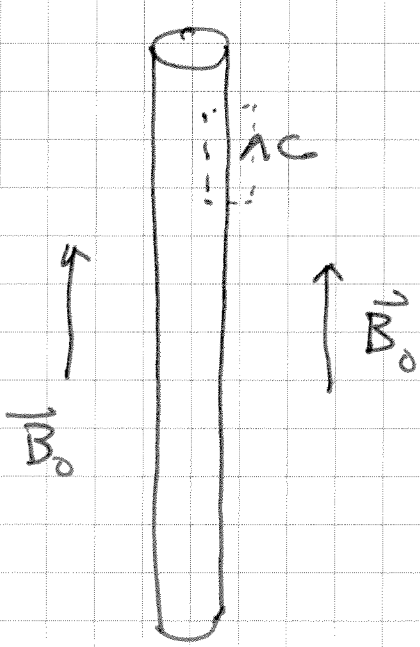
<u>Magnetization Inside</u>

$$\vec{M}_i = \chi_m \vec{H}_i$$

$$= \frac{\chi_m}{\mu_r} \frac{\vec{B}_o}{\mu_o}$$

$$= \frac{\chi_m}{1+\chi_m} \frac{\vec{B}_o}{\mu_o}$$

<u>Ex</u> Place thin cylinder in applied field



Pillbox $B_i^{\perp} = B_o^{\perp} = 0$

Amperian Path

$$\oint \vec{H} \cdot d\vec{l} = I_{f enc} = 0$$

$$\vec{H_i} = \vec{H_i^{\parallel}} = \vec{H_o^{\parallel}} = \vec{H_o}$$

$$\vec{H_o} = \frac{\vec{B_o}}{\mu_o} = \vec{H_i}$$

$$\vec{M_i} = X_m \vec{H_i} \qquad \vec{B_i} = \mu_o \mu_r \vec{H_i}$$

$$\vec{B_i} = \mu_o \mu_r \vec{H_i} = \mu_o \mu_r \left(\frac{\vec{B_o}}{\mu_o} \right) = \mu_r \vec{B_o}$$

$\Rightarrow$ Magnetic field increase by $1 + X_m$

$\Rightarrow$ Slab and needle have different effects because slab minimizes surface current and needle maximizes surface current.

<u>Ex</u> Suppose cylinder is an iron nail with $\mu_r = 1000$ and the applied field is the earth's field $B_o = 4 \times 10^{-5}\,T$

$$B_i = \mu_r B_o = 4 \times 10^{-2}\,T$$

$$\vec{M}_i = \chi_m \vec{H}_i = \chi_m \vec{H}_o = \chi_m \frac{\vec{B}_o}{\mu_o}$$

$$|\vec{M}_i| = \frac{\chi_m B_o}{\mu_o} = (1000)\left(\frac{4 \times 10^{-5}\,T}{4\pi \times 10^{-7}\,Tm/A}\right)$$

$$= 32{,}000\ A/m$$

If nail 10 cm long with radius 1 mm, the magnetic moment of the nail is

$$m = M_i V = M_i \pi r^2 h$$

$$= (32{,}000\ A/m)(0.1\,m)\,\pi\,(0.001\,m)^2$$

$$= 0.003\ A\,m^2$$

Surface Current

$$|\vec{K}_b| = |\vec{M} \times \hat{n}| = 32{,}000\, A/m$$

Total Surface Current

$$I = |K_b| h = 3200\, A.$$