Magnetic Materials

Magnetization Density ($\vec{M}$) - Magnetic dipole moment per unit volume.

Dipole Vector Potential

$$\vec{A}_{dip} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$\vec{m}$ = Magnetic dipole moment

Vector Potential of System with Magnetization $\vec{M}$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{M}(\vec{r}\,') \times \hat{r}''}{(r'')^2} \, d\tau'$$

As before, $\nabla'\left(\frac{1}{r''}\right) = \frac{\hat{r}''}{(r'')^2}$

so

$$\vec{A} = \frac{\mu_0}{4\pi} \int \vec{M}(\vec{r}\,') \times \nabla'\left(\frac{1}{r''}\right) d\tau'$$

Vector Identity

$$\nabla \times (f\vec{A}) = f(\nabla \times \vec{A}) - \vec{A} \times \nabla f$$

Let $f = \frac{1}{r''}$, $\vec{A} = \vec{M}$

$$\Rightarrow \nabla' \times \left(\frac{\vec{M}}{r''}\right) = \frac{1}{r''}(\nabla' \times \vec{M}) - \vec{M} \times \nabla'\left(\frac{1}{r''}\right)$$

$$\Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_V \vec{M}(\vec{r}\,') \times \nabla'\left(\frac{1}{r''}\right) d\tau'$$

$$= \frac{\mu_0}{4\pi}\int_V \frac{\nabla' \times M(\vec{r}\,')}{r''} d\tau' - \frac{\mu_0}{4\pi}\int_V \nabla' \times \left(\frac{\vec{M}}{r''}\right) d\tau'$$

Divergence Thm for Curl

$$\int_V \nabla \times \vec{A}\, d\tau = -\oint_S \vec{A} \times \hat{n}\, da$$

So

$$\vec{A} = \frac{\mu_0}{4\pi}\int_V \frac{\nabla' \times M(\vec{r}\,')}{r''} d\tau' + \frac{\mu_0}{4\pi}\oint_S \frac{\vec{M}(\vec{r}\,') \times \hat{n}'}{r''} da'$$

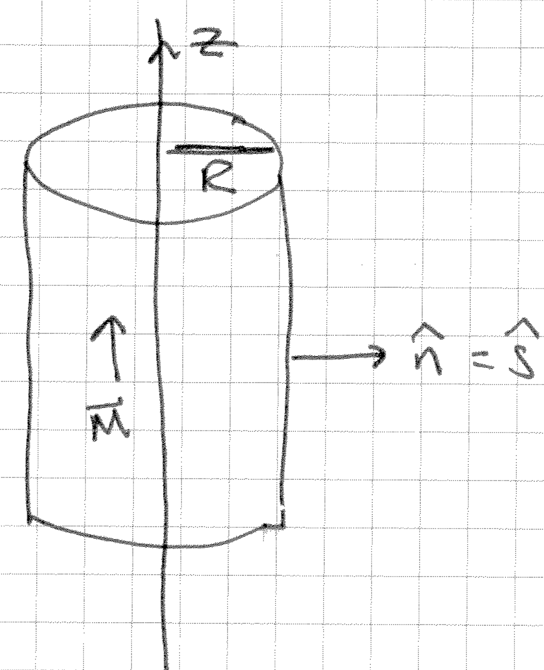
Bound Current Densities

Volume $\quad \vec{J}_b = \nabla \times \vec{M}$

Surface $\quad \vec{K}_b = \vec{M} \times \hat{n}$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}_b(\vec{r}\,')}{r''}\,d\tau' + \frac{\mu_0}{4\pi}\int_S \frac{\vec{K}_b(\vec{r}\,')}{r''}\,d\tau'$$

__Ex__ Cylinder, infinite in $\hat{z}$ direction of radius R with constant magnetization density $\vec{M} = M_0\,\hat{z}$



Bound Currents (Volume)

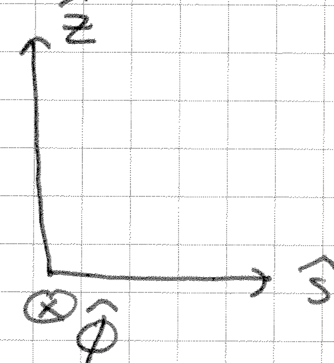
$$\vec{J}_b = \nabla \times \vec{M} = 0$$

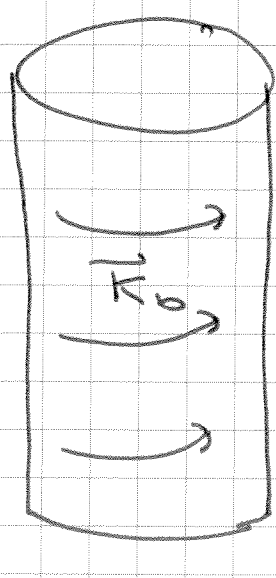
Bound Surface Current

Since the cylinder is infinitely long, only have to worry about sides.

$$\vec{K}_b = \vec{M} \times \hat{n} = (M_0 \hat{z}) \times \hat{s}$$

$$= M_0 \hat{\phi}$$





$\Rightarrow$ Once we have $\vec{J}_b$, $\vec{K}_b$ we can ignore the material.

The current density is the same as that of an infinite solenoid.

Fields

Outside : $\vec{B}_o = 0$

Inside : $\vec{B}_i = \mu_0 K_b \hat{z} = \mu_0 M_0 \hat{z}$

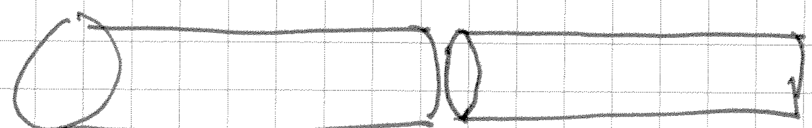
How much current? For NdFeB magnets,

$M_0 = 1.02 \times 10^6 \, A/m = K_b$.

$\Rightarrow$ If cylinder was 1m long, a total surface current of $10^6 A$ flows around its surface producing a field of

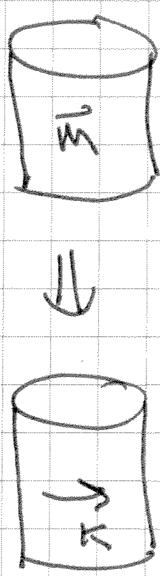
$$B = \mu_0 K_b = 1.28 T$$

$\Rightarrow$ Since $\vec{B}^{\parallel}$ is continuous, this would be the field in a small gap in a long magnet



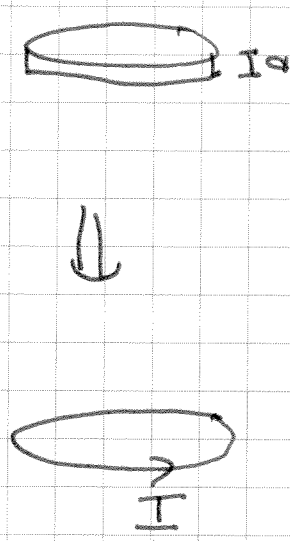
With the bound current density, we already know how to compute the field of ~~may~~ many common magnet geometries.

<u>Cylinder</u>



Finite Solenoid

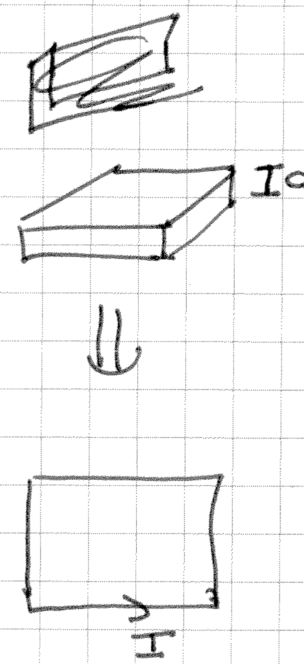
<u>Disk</u>



Ring

$I = K_b a$

<u>Wafer</u>



Squar Loop

$I = K_b a$

$\Longrightarrow$ If magnetization uniform, the total magnetic moment is $\vec{m} = \vec{M} V$ where V is the volume, so we can use dipole fields to compute field ~~far~~ from the magnet.