Maxwell's Eqns

The integral form of Maxwell's equations should now suggest thing to try.

Gauss

$$\oint_S \vec{E} \cdot d\vec{a} = \Phi_e = \frac{Q_{enc}}{\varepsilon_0}$$

No Magnetic Monopoles (Gauss' Law for Magnetism)

$$\oint \vec{B} \cdot d\vec{a} = \Phi_m = 0$$

Faraday

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{a} = -\frac{d\Phi_m}{dt}$$

Amperes

$$\oint_c \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d}{dt}\int_S \vec{E} \cdot d\vec{a}$$

$$= \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_e}{dt}$$

We will use the Divergence Thm and Stoke's Thm to turn the integral Maxwell's Eqn into the differential Maxwell's eqns. The differential equations are local and cannot depend on properties of regions like Q_{enc} or I_{enc}.

Densities

Linear Charge Density (λ) - Charge per unit length

$$\lambda = \frac{Q}{L}$$

Surface Charge Density (σ) - Charge per unit area

$$\sigma = \frac{Q}{A}$$

Volume Charge Density (ρ) - Charge per unit volume

$$\rho = \frac{Q}{V}$$

The total charge enclosed in a volume V is then

$$Q_{enc} = \int_V \rho \, d\tau$$

<u>Current Density ($\vec{J}$)</u> - The charge per unit area per unit time flowing in the direction $\hat{n}$ (the positive $\hat{n}$ direction is in the direction of flow of positive charge)

$$\vec{J} = \frac{Q}{A \cdot t} \hat{n}$$

$$\Rightarrow \quad I_{enc} = \int_S \vec{J} \cdot d\vec{a} = \frac{Q}{t}$$

flowing through S.

The other terms in Maxwell's Eqns

$\vec{E}$ - Electric Field

$\vec{B}$ - Magnetic Field

$\mu_0 \equiv 4\pi \times 10^{-7} \frac{Tm}{A}$ = Permeability of Free Space

$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$ = Permittivity of Free Space

<u>Note</u> The speed of light is now defined to be

$$c = 299,792,458 \ m/s$$

and $$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} \quad \text{exactly}$$

ε_0, μ_0, c are now defined not measured constants.

Now work on Maxwell

Gauss

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_V \rho \, d\tau$$

$$\parallel$$

Divergence Thm

$$\int_V \nabla \cdot \vec{E} \, d\tau = \frac{1}{\varepsilon_0}\int_V \rho \, d\tau$$

for all V

$$\Longrightarrow \quad \nabla \cdot \vec{E} = \rho/\varepsilon_0$$

No Magnetic Monopoles

$$\oint_S \vec{B} \cdot d\vec{a} = \int_V \nabla \cdot \vec{B} \, d\tau = 0$$

$$\Longrightarrow \quad \nabla \cdot \vec{B} = 0$$

Faraday

$$\oint_C \vec{E}\cdot d\vec{l} = \frac{d}{dt}\int_S \vec{B}\cdot d\vec{a} = \int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a}$$

$$\parallel$$

Stokes's $\displaystyle\int_S (\nabla\times\vec{E})\cdot d\vec{a}$ for all S

$$\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Ampere

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0\int_S \vec{J}\cdot d\vec{a} + \mu_0\varepsilon_0\frac{d}{dt}\int_S \vec{E}\cdot d\vec{a}$$

$$\parallel$$

Stokes $\displaystyle\oint_S (\nabla\times\vec{B})\cdot d\vec{a} =$ for all S

$$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}$$

Differential Form of Maxwell's Eqns

Gauss $\qquad \nabla \cdot \vec{E} = \rho / \varepsilon_0$

No Magnetic Monopoles $\qquad \nabla \cdot \vec{B} = 0$

Faraday $\qquad \nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$

Ampere $\qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

The magnetic field is solenoidal, $\nabla \cdot \vec{B} = 0$, so $\exists$ a field $\vec{A}$ called the <u>vector potential</u> s.t.

$$\vec{B} = \nabla \times \vec{A}$$

By second derivative rule $\nabla \cdot (\nabla \times \vec{A}) = 0$

If $\vec{B}$ is constant, the electric field is irrotational, $\nabla \times \vec{E} = 0$, therefore $\exists$ a function V called the scalar potential (or simply the potential) s.t.

$$\vec{E} = -\nabla V$$

Again by the second derivative formula

$$\nabla \times \nabla V = 0$$

If $\vec{B}$ is changing,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t} \nabla \times \vec{A}$$

$$= -\nabla \times \frac{\partial \vec{A}}{\partial t}$$

$$\Rightarrow \vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

Maxwell's Eqns give us the divergence and the curl of $\vec{E}$ and $\vec{B}$ and by Helmholtz Thm therefore uniquely define the fields

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If the magnetic field is constant, $\vec{E} = -\nabla V$

and $\nabla \cdot \vec{E} = -\nabla^2 V = \rho/\epsilon_0$

$\nabla^2 V = -\rho/\epsilon_0$ Poisson's eqn.

If $\vec{E}$ is constant,

$$\nabla \times \vec{B} = \nabla \times (\nabla \times \vec{A}) = \mu_0 \vec{J}$$

$$= \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J}$$

we can select a <u>gauge</u> where $\nabla \cdot \vec{A} = 0$ so

$$\nabla^2 \vec{A} = -\mu_0 \vec{J} \quad \text{(3 poisson's eqns)}$$

and we recover the front cover of the text and can solve everything.