Practice Final Exam - Spring 2013

Work four of the six problems. Place the problems in the order you wish them graded. The first two problems form the first half test; the second two problems form the second half test.

Problem 4.1 A wire of length 2ℓ runs along the x-axis and is centered at the origin. The wire is thinner in the middle than at the two ends. The cross-sectional area of the wire is given by $A(x) = A_0(a^2 + x^2)$, where A_0 and a are constants. Make the approximation that the current density depends only on x.

Problem 4.2 The $x - y$ plane is the boundary between two regions $z > 0$ and $z < 0$ with different electric and magnetic fields. The fields below the plane are

$$\vec{E}_- = \gamma(\hat{x} + 2\hat{y})$$

and

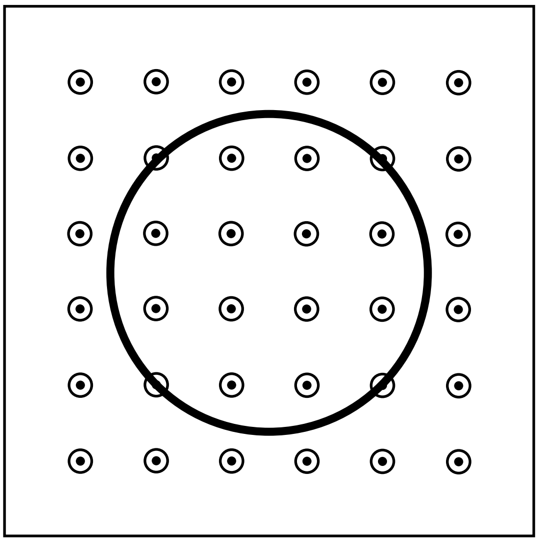
$$\vec{B}_- = \alpha(\hat{x} + \hat{z})$$

The plane has a surface charge density σ and a surface current density $\vec{K} = \Gamma\hat{y}$. Γ, γ, and α are constants. Find the electric and magnetic field above the plane ($z > 0$). Note, while this problem in electro/magnetostatic, it could be altered to include time dependent terms.

Problem 4.3 A solenoid of radius b has N turns wound over ℓ distance and carries current I. The solenoid contains a hollow cylindrical iron core with relative permeability μ_r at the operating field. The iron core has inner radius a and outer radius b. Compute $\vec{B}$, $\vec{H}$, $\vec{M}$, $\vec{J}_b$, and $\vec{K}_b$ everywhere.

Problem 4.4 A spherical system of radius a has potential $V(a, \theta) = \gamma\cos(\theta)$ at its surface. Compute the electric field inside the system.

Problem 4.5 A circular ring of conducting wire is in a region with changing magnetic field as shown below. The radius of the ring is increasing as $a(t) = a_1 t^2$, where a_1 is constant. The magnetic field is $B(t) = B_0 t^2$ in the direction drawn with B_0 constant. The conductivity of the wire is σ and it has cross-sectional area A_w. Compute the current flowing the the wire as a function of time and give its direction.



Problem 4.6 A spherical system is built with a free charge density $\rho(r) = \gamma r$ $(r < a)$ embedded in a linear dielectric material of radius a with relative permittivity ϵ_r. Calculate the potential difference between the center of the system and the surface ΔV_{0a}. γ is constant.