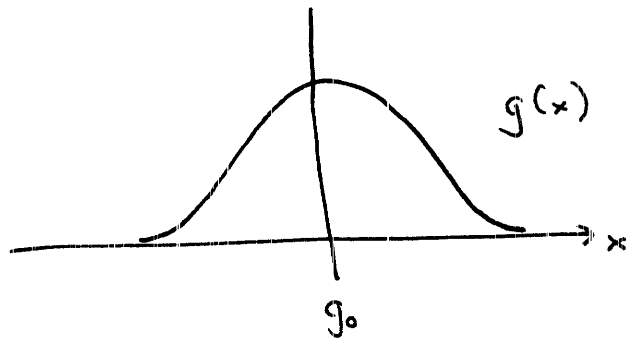
Velocity, Wavevector, and Phase

Consider a pulse whose maximum occurs at $x=0$ at $t=0$.



If the wave moves to the right with velocity v

the $f(x,t) = g(x-vt)$

We can determine v by implicit differentiation. The location of g_0 satisfies $0 = x - vt$

$$0 = dx - v\,dt$$

$$\frac{dx}{dt} = \text{pulse velocity} = v$$

If the pulse moved to the left, $f(x,t) = g(x+vt)$

$$0 = x + vt \qquad 0 = dx + v\,dt$$

$$\frac{dx}{dt} = \text{wave velocity} = -v$$

We will be interested in wave characterized by k, ω.

$$v = \omega / k$$

$$g(x - vt) = g\left(x - \frac{\omega}{k} t\right) = g'(kx - \omega t)$$

where g' absorbs the $1/k$ factor.

The speed of this wave is

$$0 = k\,dx - \omega\,dt$$

$$\frac{dx}{dt} = \text{wave speed} = \frac{\omega}{k} \quad \text{right moving}$$

Likewise,

$g'(kx + \omega t)$ is a left moving wave.

Often, the frequency of the wave does not change as it moves across a boundary. It is therefore convenient for all waves to have the same sign ωt term.

Consider, $g'(-kx - \omega t)$ $\quad 0 = -kx - \omega t$ locates g'_0

$$0 = -k\,dx - \omega\,dt$$

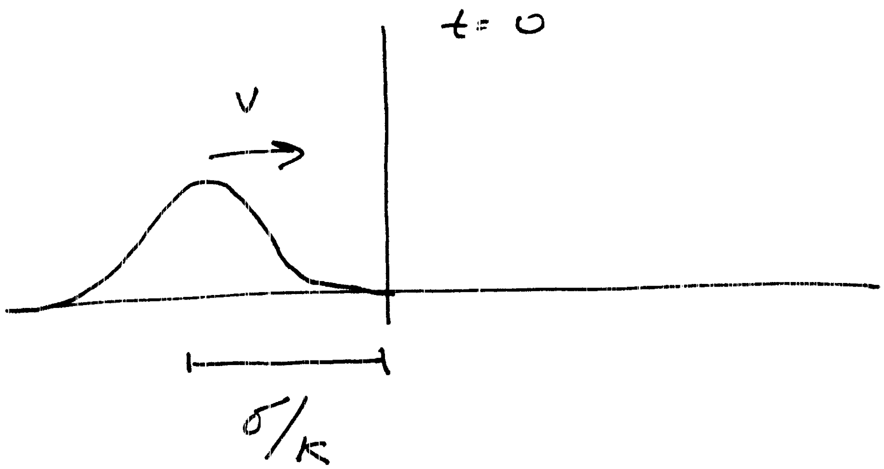
$$\frac{dx}{dt} = \text{wave velocity} = -\frac{\omega}{k} \quad \text{left travelling.}$$

We can then alway uses waves of the form

$$g(\pm kx - \omega t)$$

and let k be a <u>wave vector</u>. $+k$ gives right travelling waves, $-k$ left travelling.

Define the Phase (σ) such that σ/k is the distance the wave falls short of the origin, the distance form g_0 to 0 at $t=0$.



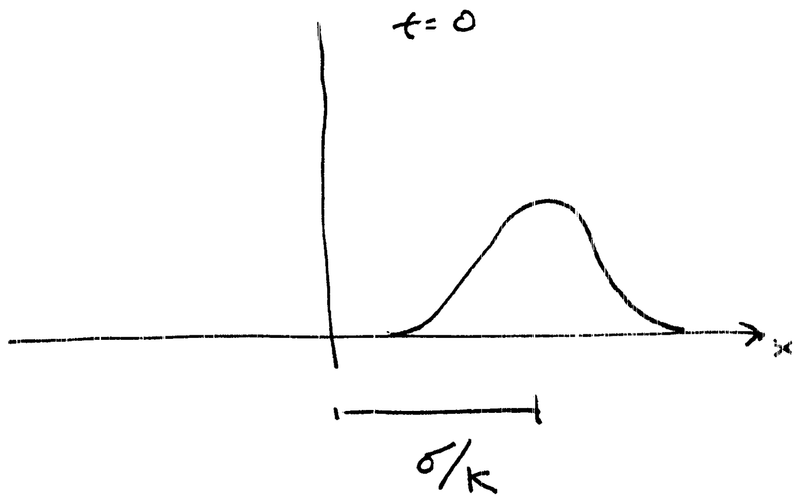
$$g\left(k\left(x+\frac{\sigma}{k}\right)-\omega t\right) = f(x,t)$$

The location of g_0 at $t=0$ is $0 = kx + \sigma$

$$x = -\sigma/k$$

$$f(x,t) = g(kx - \omega t + \sigma)$$

For left moving waves, σ/k is still the distance the wave falls short of the origin.



$$f(x,t) = g\left(-k\left(x - \sigma/k\right) - \omega t\right) = g(-kx - \omega t + \sigma)$$

At $t=0$, g_0 is at $x = \sigma/k$.

Translating this to cosine waves, where $\cos(0)$ becomes g_0 we have

Right travelling - $f(x,t) = A\cos(kx - \omega t + \sigma)$

Left travelling $f(x,t) = A\cos(-kx - \omega t + \sigma)$