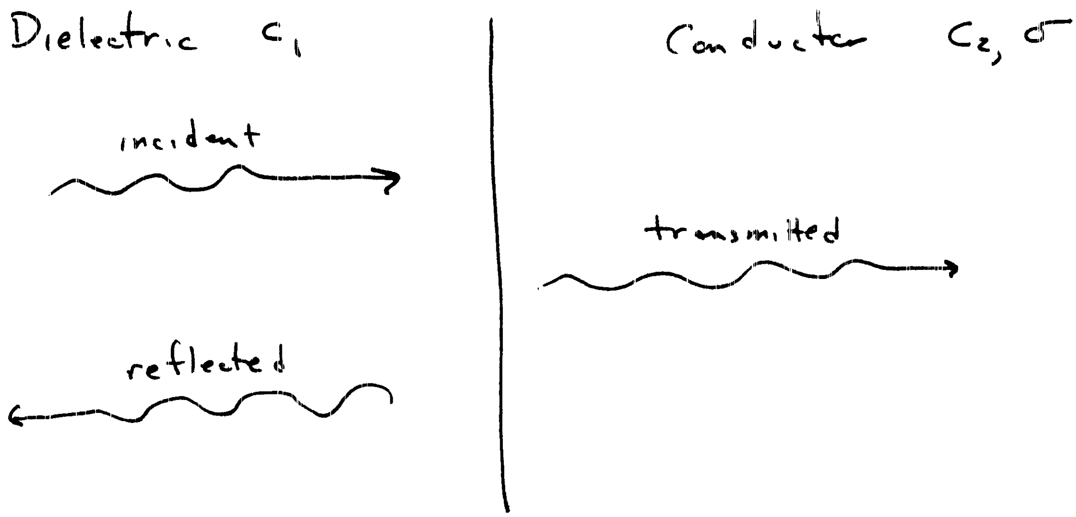
Reflection at Conducting Surface



Dielectric Solutions $z<0$

$$\vec{E}_i = E_{0i}\, e^{i(k_1 z - \omega t)}\, \hat{x}$$

$$\vec{B}_i = \frac{E_{0i}}{c_1}\, e^{i(k_1 z - \omega t)}\, \hat{y}$$

Reflected

$$\vec{E}_r = E_{0r}\, e^{i(-k_1 z - \omega t)}\, \hat{x}$$

$$\vec{B}_r = -\frac{E_{0r}}{c_1}\, e^{i(-k_1 z - \omega t)}\, \hat{y}$$

$$c_1 = \frac{\omega}{k_1} \qquad k_1 \text{ real}$$

Conductor Solutions $z>0$

$$\vec{E}_t = E_{0t}\, e^{i(k_2 z - \omega t)}\, \hat{x}$$

$$\vec{B}_t = E_{0t}\frac{k_2}{\omega}\, e^{i(k_2 z - \omega t)}\, \hat{y}$$

k_2 complex

Define $c_2 = \dfrac{\omega}{k_2}$ also complex.

With this definition, waves are identical to two dielectric interface.

Boundary Conditions (Same as always)

(1) $\epsilon_1 E_1^\perp - \epsilon_2 E_2^\perp = \sigma_f$ (2) $B_1^\perp = B_2^\perp$

(3) $\vec{E}_1^{\,\parallel} = \vec{E}_2^{\,\parallel}$

(4) $\frac{1}{\mu_1} \vec{B}_1^{\,\parallel} - \frac{1}{\mu_2} \vec{B}_2^{\,\parallel} = \vec{K}_f \times \hat{n}$

$\Rightarrow$ Since $E_1^\perp$ and $E_2^\perp = 0$, $\sigma_f = 0$.

$\Rightarrow$ $\vec{K}_f$ is a surface current, a current confined to a thin layer at the surface. Unless $\vec{E}_\parallel$ is very large immediately inside the conductor, $\vec{K}_f = 0$. This is simply the statement that we can model all currents as volume currents.

$\Rightarrow$ With these observations, we have the same wave equations, the same boundary conditions and the same ~~fictitious~~ fictitious except K_2 and our c_2 are complex.

$\Rightarrow$ The solution must be the same.

$\Rightarrow$ Define $\beta = \frac{\mu_1}{\mu_2} \frac{c_1}{c_2} = \frac{\mu_1}{\mu_2} c_1 \frac{K_2}{\omega}$

K_2 complex, now β complex.

Solution as before with B complex

$$\frac{E_{or}}{E_{oi}} = \frac{1-B}{1+B} \qquad \frac{E_{ot}}{E_{oi}} = \frac{2}{1+B}$$

where

$$B = \frac{\mu_1}{\mu_2} c_1 \frac{k_2}{\omega} = \frac{\mu_1}{\mu_2} c_1 \frac{|k_2|}{\omega} e^{i\phi}$$

Reflection Coefficient

$$R = \frac{I_r}{I_i} = \frac{\frac{1}{2}\varepsilon_1 c_1 E_{ro}^* E_{ro}}{\frac{1}{2}\varepsilon_1 c_1 E_{io}^* E_{io}}$$

$$= \frac{(1-B^*)(1-B)}{(1+B^*)(1+B)} = \frac{1-(B+B^*)+|B|^2}{1+(B+B^*)+|B|^2}$$

Let $B_0 = \frac{\mu_1}{\mu_2} c_1 \frac{|k_2|}{\omega}$ real

$$B = B_0 e^{i\phi}$$

$$B + B^* = B_0 \cos\phi + i B_0 \sin\phi + B_0 \cos\phi - i B_0 \sin\phi$$

$$= 2B_0 \cos\phi$$

$$R = \frac{1 - 2B_0 \cos\phi + B_0^2}{1 + 2B_0 \cos\phi + B_0^2}$$

$$|k_2| = \omega \sqrt{\varepsilon\mu \sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2}}$$

$$\cancel{\frac{\omega^2}{c_2^2}\left\{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2\right\}^{1/4}}$$

As $\sigma \to \infty$, $|k_2| \to \infty$, $B \to \infty$, $R \to 1$

All the wave is reflected by a perfect conductor, which is why metals make good mirrors.

Transmission Coefficient

$$T = 1 - R = \frac{4 B_0 \cos\phi}{1 + 2B_0 \cos\phi + B_0^2}$$

$\Rightarrow$ this is the part of the wave that passes through the interface, all this wave is ultimately absorbed.

$\Rightarrow$ Wave speed

$$c_2 = \frac{\omega}{k_r} = \frac{1}{\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2} + 1\right]^{1/2}}$$