Electricity and Magnetism - Test 1 - Spring 2013

Work four of the six problems. Place the problems in the order you wish them graded. The first two problems form the first half test; the second two problems form the second half test.

Problem 1.1 Consider the following electromagnetic field

$$\vec{E}(x,y,z) = E_0 \sin(kx - \omega t)\hat{y}$$

$$\vec{B}(x,y,z) = B_0 \sin(kx - \omega t)\hat{z}$$

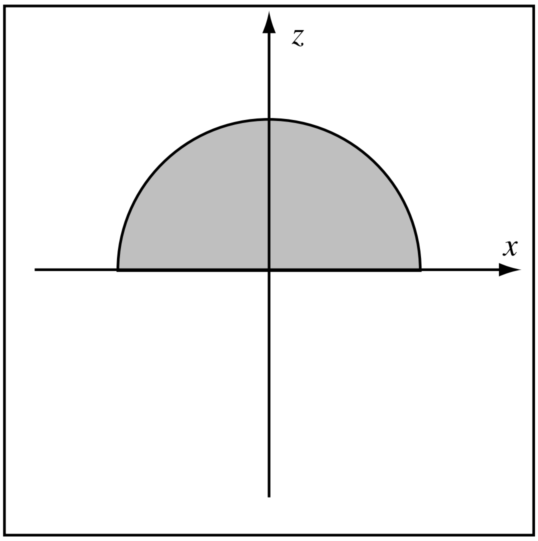
where E_0, B_0, k, and ω are constants which may be related. The field exists in a region of space where both the charge density ρ and the current density $\vec{J}$ are zero. Does this field satisfy Maxwell's equations? If it does not, state all of the equations that are not satisfied for any non-zero choice of E_0, B_0, k, and ω. If the fields do satisfy Maxwell's equations, what algebraic equations must be satisfied by E_0, B_0, k, and ω for Maxwell's equations to be satisfied?

Problem 1.2 As a first model for the charge density of a semiconductor diode we could use

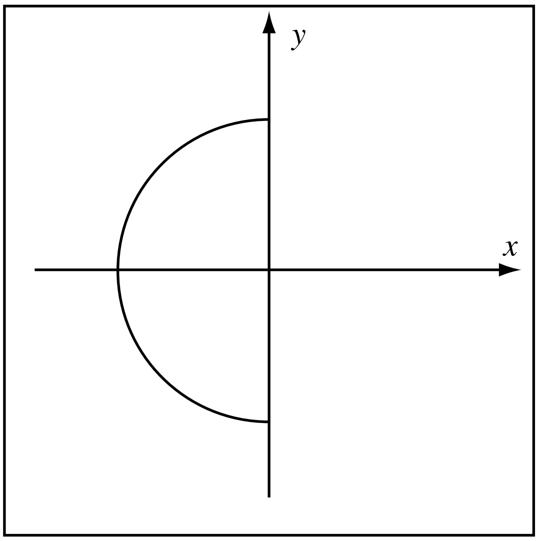
$$\rho(x) = \frac{\rho_0}{a} x \exp(-(x/a)^2).$$

Sketch this charge density and the electric field. Calculate the electric field everywhere.

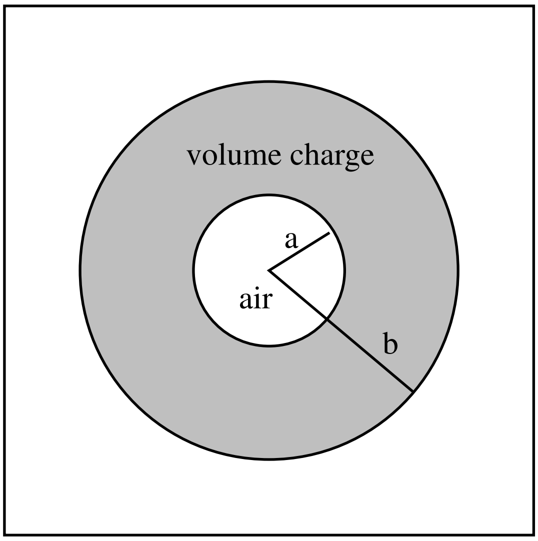
Problem 1.3 Compute the electric field at the origin of a uniformly charged half-sphere with charge density ρ where ρ is non-zero at points $r < a$ and $z > 0$.



Problem 1.4 Compute the electric potential at a point P along the z axis, $\vec{r}_P = (0,0,z)$, of a half-circle with constant linear charge density λ. The half-circle lies in the $x - y$ plane, has radius a, and is non-zero for $\pi/2 < \phi < 3\pi/2$.



Problem 1.5 A NON-UNIFORM spherical volume charge has charge density $\rho(r) = \gamma r$ for $a < r < b$ and zero otherwise; γ is a constant. Calculate the electric field and electric potential everywhere. Let $V(\infty) = 0$.



Problem 1.6 A capacitor is formed of two concentric spherical conductors. The inner conductor has radius a and the outer conductor has inner radius b and outer radius c. Compute the capacitance between the inner and outer conductors. Compute the energy stored in the capacitor if the inner conductor has charge Q and the outer conductor charge $-Q$ in two ways: (1) from the capacitance, (2) from the energy density of the fields.