# Helmholtz Thm

If a vector field goes to zero at $\infty$, the field is uniquely determined by its divergence and curl.

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## Curl Free Fields $\quad \nabla \times \vec{A} = 0 \implies$

- $\int_{\vec{r}_a \to \vec{r}_b} \vec{A} \cdot d\vec{l}$ is independant of path
- $\oint \vec{A} \cdot d\vec{l} = 0$ for closed loop
- $\vec{A} = \nabla f$ for some $f$
- The field is irrotational

## Divergenceless Fields

If $\nabla \cdot \vec{A} = 0 \Rightarrow$

- $\vec{A} = \nabla \times \vec{F}$ for some $\vec{F}$
- $\int_S \vec{A} \cdot d\vec{a}$ is independent of $S$ for a given bounding curve $C$.
- $\oint_S \vec{A} \cdot d\vec{a} = 0$ for all closed $S$
- The field is solenoidal

Note, the second derivative identities

$$\nabla \cdot (\nabla \times \vec{A}) = 0 \quad \text{and} \quad \nabla \times (\nabla f) = 0$$

guarantee that if $\vec{A} = \nabla \times \vec{F}$ the $\nabla \cdot \vec{A} = 0$ and if $\vec{A} = \nabla f$ then $\nabla \times \vec{A} = 0$