Homework 7

Due Tuesday April 8th 2014 - at beginning of class

Griffiths' 4 Problems

4.39 (Griffiths 3rd edition problem 4.36)

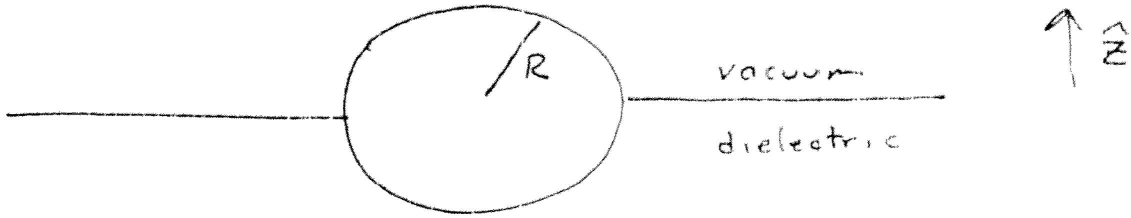
5.4

5.5

5.9

5.11

(4.39)



(a) $V = \dfrac{V_0 R}{r}$ if potential same as missing dielectric.

<u>Field everywhere</u>

$$\vec{E} = -\nabla V = \frac{V_0 R}{r^2}\,\hat{r}$$

<u>Polarization</u> $\vec{P} = 0, \quad z > 0$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} = \frac{\varepsilon_0 V_0 R \chi_e}{r^2}\,\hat{r} \qquad z < 0$$

<u>Bound Charge</u>

$$\rho_b = -\nabla \cdot \vec{P} = 0$$

$$\sigma_b = \vec{P} \cdot \hat{z} = 0 \quad \text{interface between planes}$$

$$\sigma_b = -\vec{P} \cdot \hat{n}\Big|_R = -\frac{\varepsilon_0 V_0 \chi_e}{R} \quad \text{at surface of sphere.}$$

– for inward normal

Free Charge

$z > 0$ Using Gaussian Pillbox

$$\Phi = \vec{E}_c(R) \cdot \hat{n} A - 0 = \frac{\sigma_{+c} A}{\varepsilon_0}$$

$$\sigma_{c+} = \varepsilon_0 \vec{E}_0(R) = \frac{\varepsilon_0 V_0}{R}$$

$z < 0$ Pillbox encloses σ_{c-} and σ_b

$$\Phi = \vec{E}_R(R) \cdot \hat{r} A - 0 = \frac{(\sigma_{c-} + \sigma_b) A}{\varepsilon_0}$$

$$\varepsilon_0 \left(\frac{V_0 R}{R^2} \right) = \sigma_{c-} + \sigma_b$$

$$\sigma_{c-} = \frac{\varepsilon_0 V_0}{R} + \frac{\varepsilon_0 V_0 \chi_e}{R}$$

$$= \frac{\varepsilon_0 V_0}{R} \varepsilon_r$$

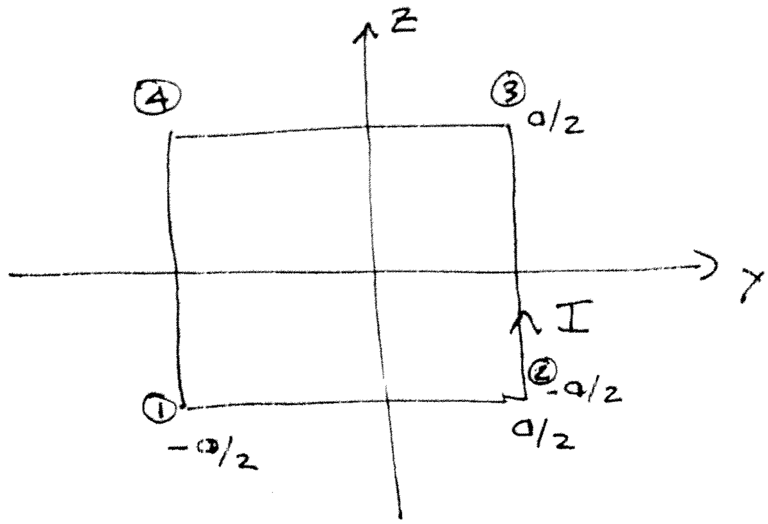
(b) The total charge is σ_{c+} over the entire sphere because $\sigma_{c+} = \sigma_{c-} + \sigma_b$, which does produce the field.

(c) Since we meet the boundary conditions, we must have the solution.

(d) (b) yes, (a) no. We would not meet the electrostatic boundary condition at the boundary.

5.4

$$\vec{B} = kz\hat{x}$$



$$\vec{F}_{12} = I\vec{L}\times\vec{B} = I(a\hat{y})\times\left(-\frac{a}{2}k\hat{x}\right)$$

$$= \frac{Ia^2}{2}k\hat{z}$$

$$\vec{F}_{23} = \int_{-a/2}^{a/2} I(a\hat{z})\times(kz\hat{x})\,dz = 0$$

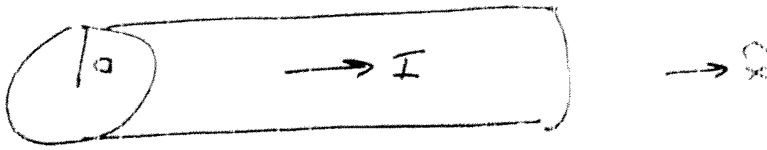
since integration of odd function over even range.

$$= \vec{F}_{41}$$

$$\vec{F}_{34} = I(-a\hat{y})\times\left(\frac{a}{2}k\hat{x}\right) = \frac{Ia^2}{2}k\hat{z}$$

$$\vec{F}_{total} = \sum\vec{F} = \vec{F}_{12} + \vec{F}_{41} = kIa^2\hat{z}$$

5.5



(a) $\vec{K} = \frac{I}{2\pi a}\hat{x} = \frac{I}{\text{circumference}}\hat{x}$

(b) $\vec{J} = \frac{J_0}{s}\hat{x}$

$$I = \int J\,da = \int_0^{2\pi} d\phi \int_0^{a} s\,ds \left(\frac{J_0}{s}\right)$$

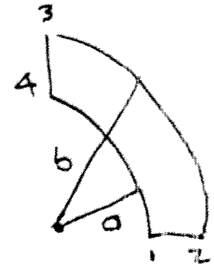
$$= 2\pi J_0 \int_0^{a} ds = 2\pi J_0 a$$

$$J_0 = \frac{I}{2\pi a}$$

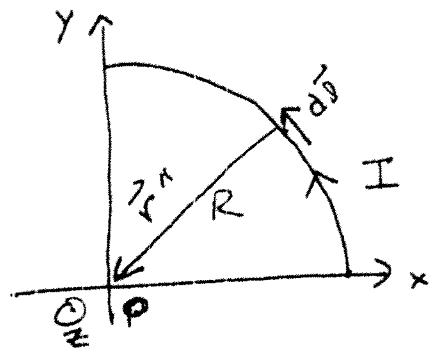
$$\vec{J} = \frac{J_0}{s}\hat{x} = \frac{I}{2\pi a s}\hat{x}$$

5.9

(a)



Magnetic field of quarter circle



Biot-Savart

$$\vec{B}_P = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}''}{r''^2}$$

$$d\vec{l}' = R\,d\phi'\,\hat{\phi}' \qquad r'' = R$$

$$|d\vec{l}' \times \hat{r}''| = |d\vec{l}'||\hat{r}''| \sin 90$$

$$= R\,d\phi'$$

$$d\vec{l} \times \hat{r}'' = R\,d\phi'\,\hat{z} \qquad (RHR)$$

$$\vec{B}_p = \frac{\mu_0 I}{4\pi} \frac{R\hat{z}}{R^2} \int_0^{\pi/2} d\phi'$$

$$= \frac{\mu_0 I}{8R} \hat{z}$$

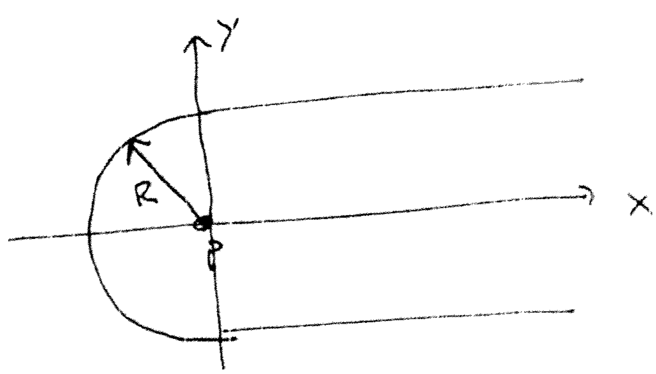
Total Field

$$\vec{B}_p = \vec{B}_{12} + \vec{B}_{23} + \vec{B}_{34} + \vec{B}_{41}$$

$$= 0 - \frac{\mu_0 I}{8b}\hat{z} + 0 + \frac{\mu_0 I}{8a}\hat{z}$$

$$= \frac{\mu_0 I}{8}\left(\frac{1}{a} - \frac{1}{b}\right)$$

(b)



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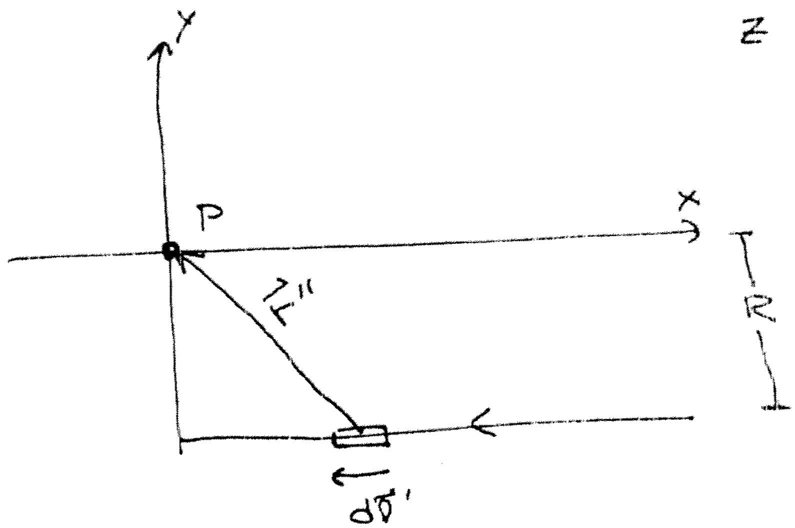
By similar calculation, the field of the half circle is

$$\frac{\mu_0 I}{4R}(-\hat{z})$$

The fields of the two lines are equal and must be calculated by integration.



$\vec{r}_P = (0,0,0)$ $\qquad$ $d\vec{l}' = -dx'\hat{x}$ (integrating $0 \to \infty$)

$\vec{r}' = (x', -R, 0)$

$\vec{r}'' = \vec{r}_P - \vec{r}' = (-x', R, 0)$

$r'' = \sqrt{x'^2 + R^2}$

$$\vec{B}_P = \frac{\mu_0 I}{4\pi} \int_0^\infty \frac{d\vec{l}' \times \vec{r}''}{r''^3}$$

$$d\vec{l}' \times \vec{r}'' = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -dx' & 0 & 0 \\ -x' & R & 0 \end{vmatrix} = -dx' R \hat{z}$$

$$\vec{B}_p = -\frac{\mu_0 I R}{4\pi}\hat{z} \underbrace{\int_0^\infty \frac{dx'}{(x'^2+R^2)^{3/2}}}_{1/R^2}$$

$$= \frac{-\mu_0 I R}{4\pi R^2}\hat{z}$$

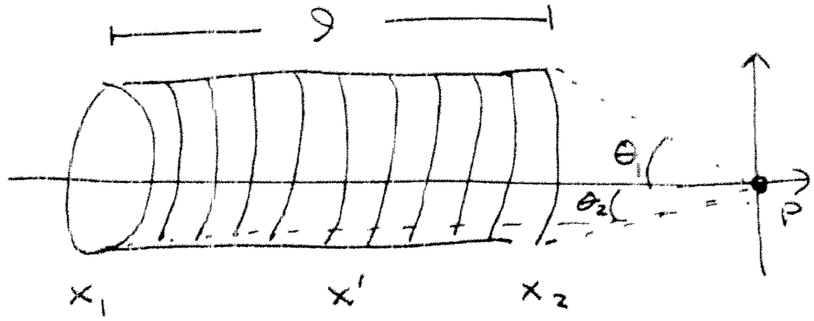
$$= \frac{-\mu_0 I}{4\pi R}\hat{z} \equiv \vec{B}_{wire,\ bottom} = \vec{B}_{wire,\ top}$$

$$\vec{B} = \vec{B}_{circle} + \vec{B}_{wire,\ bottom} + B_{wire,\ top}$$

$$= -\hat{z}\left(\frac{\mu_0 I}{4R} + 2\cdot\frac{\mu_0 I}{4\pi R}\right)$$

$$= -\hat{z}\frac{\mu_0 I}{4R}\left(1 + \frac{2}{\pi}\right)$$

5.11



$$K = I\frac{N}{l}$$

Let x' be location of one of the rings

$$dI = K\,dx' = I\frac{N}{l}\,dx'$$

$$\vec{B}_p = \int dB = \int_{x_1}^{x_2} \frac{\mu_0 dI}{2}\frac{a^2}{(a^2+x'^2)^{3/2}}\hat{x}$$

$$= \mu_0 \frac{N}{l}\frac{I}{2}a^2 \underbrace{\int_{x_1}^{x_2}\frac{dx'}{(a^2+x'^2)^{3/2}}}\hat{x}$$

$$= \frac{\mu_0 N I a^2 \hat{x}}{2l}\left[\frac{x}{a^2\sqrt{x^2+a^2}}\right]_{x_1}^{x_2}$$

$$= \frac{1}{2}\mu_0 \frac{N}{l} I \hat{x}\left(\frac{x_2}{\sqrt{x_2^2+a^2}} - \frac{x_1}{\sqrt{x_1^2+a^2}}\right)$$

$$x_1 = -a\cos\theta_2 \qquad x_2 = -a\cos\theta_1$$

$$\frac{x_1}{\sqrt{x_1^2+a^2}} = -\cos\theta_2$$

$$\frac{x_2}{\sqrt{x_2^2+a^2}} = -\cos\theta_1$$

$$\vec{B}_p = \frac{1}{2}\mu_0 \frac{N}{l} I \hat{x} \left(\cos\theta_2 - \cos\theta_1 \right)$$

If solenoid becomes long, $\theta_1 \to \pi$, $\theta_2 \to 0$

$$(\ ) \to 2$$

$$\vec{B}_p = \mu_0 \frac{N}{l} I \hat{x} \quad \checkmark$$